European Journal of Operational Research 226 (2013) 615–625

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

The extended QUALIFLEX method for multiple criteria decision analysis based on interval type-2 fuzzy sets and applications to medical decision making

Ting-Yu Chen [a,*], Chien-Hung Chang [b,1], Jui-fen Rachel Lu [c,2]

[a] Department of Industrial and Business Management, Graduate Institute of Business and Management, College of Management, Chang Gung University, 259, Wen-Hwa 1st Road, Kwei-Shan, Taoyuan 333, Taiwan

[b] Stroke Center and Stroke Section, Department of Neurology, Linkou Medical Center, Chang Gung Memorial Hospital, 5, Fusing Street, Gueishan, Taoyuan 333, Taiwan and Department of Electrical Engineering, College of Engineering, Chang Gung University, 259, Wen-Hwa 1st Road, Kwei-Shan, Taoyuan 333, Taiwan

[c] Department of Health Care Management, College of Management, Chang Gung University, 259, Wen-Hwa 1st Road, Kwei-Shan, Taoyuan 333, Taiwan

ARTICLE INFO

Article history:
Received 6 December 2011
Accepted 20 November 2012
Available online 29 November 2012

Keywords:
Decision analysis
QUALIFLEX
Outranking method
Multiple criteria decision-making
Interval type-2 fuzzy set

ABSTRACT

QUALIFLEX, a generalization of Jacquet-Lagreze's permutation method, is a useful outranking method in decision analysis because of its flexibility with respect to cardinal and ordinal information. This paper develops an extended QUALIFLEX method for handling multiple criteria decision-making problems in the context of interval type-2 fuzzy sets. Interval type-2 fuzzy sets contain membership values that are crisp intervals, which are the most widely used of the higher order fuzzy sets because of their relative simplicity. Using the linguistic rating system converted into interval type-2 trapezoidal fuzzy numbers, the extended QUALIFLEX method investigates all possible permutations of the alternatives with respect to the level of concordance of the complete preference order. Based on a signed distance-based approach, this paper proposes the concordance/discordance index, the weighted concordance/discordance index, and the comprehensive concordance/discordance index as evaluative criteria of the chosen hypothesis for ranking the alternatives. The feasibility and applicability of the proposed methods are illustrated by a medical decision-making problem concerning acute inflammatory demyelinating disease, and a comparative analysis with another outranking approach is conducted to validate the effectiveness of the proposed methodology.

1. Introduction

Paelinck (1976) generalized Jacquet-Lagreze's permutation method to develop the flexible multiple criteria decision-making method, known as QUALIFLEX. The methodology of QUALIFLEX originates from the outranking model, which arranges a set of preference rankings that best satisfies a given concordance measure (Hwang and Yoon, 1981). The QUALIFLEX method approaches multiple criteria decision-making problems by testing how each possible ranking of alternatives is supported by different criteria (Lahdelma et al., 2003). The main advantage of the QUALIFLEX method is the correct treatment of cardinal and ordinal information (Rebai et al., 2006). QUALIFLEX enables rankings of several criteria while also ranking the relative importance of each criterion (Sarabando and Dias, 2010). Several useful extensions have been developed to enhance the QUALIFLEX method. For example, Griffith and Paelinck (2011) extended QUALIFLEX to develop a qualitative regression method, known as QUALIREG, which derives an optimal ranking for the given rankings and criteria weights. Chen and Wang (2009) extended Jacquet-Lagreze's permutation method to develop a new fuzzy permutation method for multiple criteria decision analysis based on interval-valued fuzzy sets.

This paper leverages Paelinck's QUALIFLEX method to develop an extended QUALIFLEX method for interval type-2 fuzzy sets (IT2FSs). IT2FSs, also known as interval-valued fuzzy sets (Zadeh, 1975), have membership values of crisp intervals within [0,1], and the concept of IT2FSs is an extension of type-1 fuzzy sets. However, it is common for real-world decision-makers to use linguistic variables to evaluate the ratings of various criteria alternatives (Chen, 2000). The concept of linguistic variables is useful in dealing with complex or ill-defined situations. Linguistic values are frequently represented by fuzzy numbers. In a similar manner, this paper employs the widely used trapezoidal form of fuzzy numbers to transform linguistic variables in an IT2FS environment. In this paper, interval type-2 trapezoidal fuzzy numbers (IT2TrFNs) are used within an IT2FS framework to propound the extended QUALIFLEX method for handling multiple criteria decision-making problems. Multiple criteria decision-making methods have been successfully used to assist medical decision making (Liberatore and Nydick, 2008). To demonstrate the feasibility of the extended

* Corresponding author. Tel.: +886 3 2118800x5678; fax: +886 3 2118500.
E-mail addresses: tychen@mail.cgu.edu.tw (T.-Y. Chen), cva9514@gmail.com (C.-H. Chang), rachel@mail.cgu.edu.tw (J.-f. Rachel Lu).
[1] Tel.: +886 3 3281200x8340; fax: +886 3 3288849.
[2] Tel.: +886 3 2118800x5483; fax: +886 3 2118345/2118186.

0377-2217/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ejor.2012.11.038

QUALIFLEX method, a real-world case study is analyzed to explore medical decision making at Chang Gung Memorial Hospital in Taiwan. Lee and Lin (2010) demonstrated that fewer than 5% of patients prefer a purely passive role in medical decision making. In addition, the patients are able to make decisions and choices regarding what they need and want (Lutz and Bowers, 2000). However, despite patient participation in medical decision making, patient information and skills may be inadequate to contribute significantly to shared decision making and ultimately to clinical outcomes (Lee and Lin, 2010). Considering the patient point of view and circumstances in the patient-centered decision-making process, the patient's opinions and judgments are inherently imprecise and involve many uncertainties. Therefore, this paper uses IT2TrFNs to capture imprecise or uncertain therapeutic information in medical decision-making analysis.

Fuzzy set theory has already been successfully applied in the fields of medical or healthcare decision making. For example, Tsai et al. (2010) evaluated healthcare organization performance using fuzzy AHP (Analytic Hierarchy Process) and fuzzy sensitive analysis-based approaches. According to the principles of fuzzy measures and fuzzy integrals, Dursun et al. (2011) developed a fuzzy multiple criteria group decision-making method to evaluate healthcare waste disposal alternatives. Based on a fuzzy inference technique, Esposito et al. (2011) proposed an evolutionary-fuzzy decision support system for assessing the health status of subjects affected by multiple sclerosis during the disease progression through time. Tartarisco et al. (2012) developed clinical decision support systems in personal health systems based on an autoregressive model, artificial neural networks, and fuzzy logic modeling. Koulouriotis and Mantas (2012) employed neural networks and hybrid neural fuzzy system to forecast health products sales. Büyüközkan and Çifçi (2012) conducted a strategic analysis of electronic service quality in the healthcare industry using a combined fuzzy AHP and fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method. Dekker (2012) indicated that healthcare is sensitive to the social complexities of the workplace, including power, gender, hierarchy and fuzzy system boundaries. This researcher used the signal detection paradigm to account for a healthcare case with multiple clinical decision-makers. Yucel et al. (2012) proposed a fuzzy risk assessment model for new healthcare information technology using a fuzzy inference system. This group integrated possible risk factors into the decision-making process of risk assessment.

Note that all of the aforementioned studies applied type-1 fuzzy set theory to the medical or healthcare areas. Because human judgment is often vague under many conditions, the available information is often insufficient to define the degree of membership precisely for certain elements. The computation associated with IT2FSs is manageable (Mendel et al., 2006); thus, IT2FSs are the most widely used of the higher order fuzzy sets because of their relative simplicity (Wu and Mendel, 2007). The advantage of IT2FSs over type-1 fuzzy sets is their ability to model second-order uncertainties (Greenfield et al., 2009). Therefore, in this paper, instead of type-1 fuzzy sets, we use IT2TrFNs in IT2FS theory to develop new medical decision-making methods to address additional imprecision and uncertainty issues.

In general, multiple criteria decision-making methods have been widely applied in the domain of medical and healthcare decision making. Liberatore and Nydick (2008) reviewed applications of AHP in medical and healthcare decision making. These researchers indicated that AHP is useful for the shared decision making between patient and doctor, the evaluation and selection of therapies and treatments, and the evaluation of healthcare technologies and policies. Grosan et al. (2008) proposed a multiple criteria procedure and applied an evolutionary scheme to medical diagnosis and treatments. Šušteršič et al. (2009) used a hierarchical multiple attribute decision model to evaluate patients' health. Tamanini et al. (2009) developed a multiple criteria model for aiding in decision making on the diagnosis of Alzheimer's disease. Mehrotra and Kim (2011) presented a multiple criteria robust weighted sum model to solve a group decision-making problem of outcomes-based budget allocations to chronic disease prevention programs. Moreno et al. (2010, 2012) dealt with medical treatment comparison from the cost-effectiveness viewpoint. Moreno et al. (2012) handled the decision problem of choosing an optimal medical treatment. This group presented regression models for the treatment cost and effectiveness and proposed an objective Bayesian variable selection procedure for choosing subsets of influential covariates. Creemers et al. (2012) established a model that allows hospital decision-makers to assess the impact of the allocation of operating room capacity on the waiting time of different classes of patients. Brailsford et al. (2012) described a simulation model for screening for breast cancer that included behavioral factors to model women's decisions on whether to attend for mammography. Starting from that decisional process and taking the nation of Italy as the population, Ippoliti and Falavigna (2012) developed an operational research study to support the (positive) role of pharmaceutical clinical research in the patient mobility process.

The objective of this work is to develop an extended QUALIFLEX method based on IT2TrFNs for dealing with medical decision-making problems in the interval type-2 fuzzy context. As stated earlier, a considerable number of valuable applications have been treated by multiple criteria decision-making methodologies in the fields of medical and healthcare decision making. In contrast, few studies have been conducted on the QUALIFLEX method in medical and healthcare applications. Thus, we extend the QUALIFLEX method in the IT2TrFN environment to address multiple criteria decision-making problems. On the other hand, many useful methods have been developed to enrich outranking decision-making methodologies, such as ELECTRE (ELimination Et Choice Translating Reality, Benayoun et al., 1966), PROMETHEE (Preference Ranking Organization METHods for Enrichment Evaluations, Brans et al., 1984), and THESEUS (Fernandez and Navarro, 2011), among others. ELECTRE easily solves the commensurateness problem by making pairwise comparisons (Merad et al., 2013). Various ELECTRE models have been developed, and ELECTRE methods have experienced a widespread and extensive use in real-world situations (Figueira et al., 2005). Therefore, to illustrate the advantages of the proposed QUALIFLEX method, we extend the ELECTRE method to the decision environment of IT2FSs to conduct a comparative study on the same medical decision problem.

This paper is organized as follows. Section 2 introduces the transformation of linguistic variables to IT2TrFNs and formulates a multiple criteria decision-making problem within an IT2TrFN framework. Section 3 develops an extended QUALIFLEX method using the concept of signed distances between IT2TrFNs. Section 4 demonstrates the feasibility and applicability of the proposed methodology by applying the proposed method to a medical decision-making problem concerning acute inflammatory demyelinating disease and conducting a comparative analysis with the widely used ELECTRE method. Section 5 presents our conclusions.

2. Multiple criteria decision environment with IT2TrFNs

The concepts of IT2FSs and IT2TrFNs are used extensively throughout this paper. Therefore, selected relevant definitions and operations of IT2FSs and IT2TrFNs are briefly reviewed in Appendix A. In addition, Appendix B provides a table of mathematical notation for ease of reference. This section establishes a decision environment based on IT2TrFNs for multiple criteria decision-making problems in which the importance weights of the criteria and the alternative criterion ratings take the form of uncertain linguistic variables.

Consider the following multiple criteria decision-making problem: Define the alternative set $A = \{A_1, A_2, \ldots, A_m\}$ consisting of m non-inferior alternatives, and the criterion set $X = \{x_1, x_2, \ldots, x_n\}$. Note that the non-inferior alternative is named differently by various disciplines: the non-dominated alternative or efficient solution in multiple criteria decision analysis, the admissible alternative in statistical decision theory, and the Pareto-optimal solution in economics (Hwang and Yoon, 1981). A feasible alternative is non-inferior if there is no other feasible alternative that will yield an improvement in one criterion without causing degradation in at least one other criterion. Each non-inferior alternative is evaluated on each of the n criteria, and the assessment is expressed as an IT2TrFN rating. The preference information of the n criteria can also be expressed as IT2TrFN weights of criterion importance.

It may be difficult to use a direct method to collect IT2TrFN data. Therefore, the alternative criterion ratings and the criterion importance weights are transformed into linguistic variables to overcome the difficulty of data collection. This paper adopted the nine-point rating scale to measure the variability in responses and obtain better sensitivity. There are nine translations of the linguistic terms into IT2TrFNs, including lower trapezoidal fuzzy numbers (Chen and Chen, 2009) and upper trapezoidal fuzzy numbers (Chen and Chen, 2008, 2009; Wei and Chen, 2009). The height of the upper trapezoidal fuzzy numbers was designated as 1.0 (Chen and Lee, 2010a,b; Chen, 2011a,b, 2012; Chen et al., 2012; Gilan et al., 2012; Wang et al., 2012), which is the same as that of ordinary normalized fuzzy numbers. However, various methods exist to designate the height of the lower trapezoidal fuzzy numbers, including 0.75 and 1.0 for five-point rating scales (Chen and Lee, 2010b); 0.9 for seven-point scales (Chen and Lee, 2010a; Wang et al., 2012); 0.9 and 1.0 for seven-point scales (Chen et al., 2012); 0.53, 0.58, 0.64, 0.70, and 1.0 for seven-point scales (Gilan et al., 2012); 0.8 and 1.0 for nine-point scales (Chen, 2011a,b, 2012); and 0.53, 0.56, 0.58, 0.64, 0.65, and 1.0 for nine-point scales (Gilan et al., 2012). In this paper, the height of the lower trapezoidal fuzzy numbers was designated as 0.8, according to Wei and Chen (2009), except for the absolutely low/high (AL/AH) and absolutely poor/good (AP/AG) responses. The AL/AH and AP/AG responses are exact without any assumption of indeterminacy; thus, the height of their corresponding lower trapezoidal fuzzy numbers was modified to 1.0 (Chen, 2011a,b, 2012). The linguistic variables and their corresponding IT2TrFNs for the importance weights and the alternative ratings are shown in Table 1.

The decision-maker evaluates the alternatives to each criterion using the linguistic terms. The linguistic ratings are expressed as corresponding non-negative IT2TrFNs based on the transformation standard. The criterion value takes the form of an IT2TrFN, and the alternative $A_i \in A$ is evaluated with respect to the criterion $x_j \in X$. Let A_{ij}^L and A_{ij}^U denote the lower and upper extremes of the IT2TrFN A_{ij}. The evaluation of the alternative A_i on the criterion x_j is expressed as follows:

$$A_{ij} = \left[A_{ij}^L, A_{ij}^U\right] = \left[\left(a_{1ij}^L, a_{2ij}^L, a_{3ij}^L, a_{4ij}^L; h_{ij}^L\right), \left(a_{1ij}^U, a_{2ij}^U, a_{3ij}^U, a_{4ij}^U; h_{ij}^U\right)\right], \quad (1)$$

where $0 \leqslant a_{1ij}^L \leqslant a_{2ij}^L \leqslant a_{3ij}^L \leqslant a_{4ij}^L \leqslant 1$, $0 \leqslant a_{1ij}^U \leqslant a_{2ij}^U \leqslant a_{3ij}^U \leqslant a_{4ij}^U \leqslant 1$, $0 \leqslant h_{ij}^L \leqslant h_{ij}^U \leqslant 1$, $a_{1ij}^U \leqslant a_{1ij}^L$, $a_{4ij}^L \leqslant a_{4ij}^U$, and $A_{ij}^L \subset A_{ij}^U$. Decision matrix D with IT2TrFNs is constructed as follows:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix} \end{array}. \quad (2)$$

The characteristics of the alternative A_i can be represented by the IT2TrFN as follows:

$$A_i = \left\{\left\langle x_j, \left[A_{ij}^L, A_{ij}^U\right]\right\rangle | x_j \in X,\ j = 1, 2, \ldots, n\right\}, \quad i = 1, 2, \ldots, m. \quad (3)$$

Non-negative IT2TrFNs can be used to express the importance weights for various decision criteria during the decision-maker's evaluation process. Based on the linguistic terms, the importance weights of criterion x_j can be expressed as follows:

$$W_j = \left[W_j^L, W_j^U\right] = \left[\left(w_{1j}^L, w_{2j}^L, w_{3j}^L, w_{4j}^L; h_j^L\right), \left(w_{1j}^U, w_{2j}^U, w_{3j}^U, w_{4j}^U; h_j^U\right)\right], \quad (4)$$

where $0 \leqslant w_{1j}^L \leqslant w_{2j}^L \leqslant w_{3j}^L \leqslant w_{4j}^L \leqslant 1$, $0 \leqslant w_{1j}^U \leqslant w_{2j}^U \leqslant w_{3j}^U \leqslant w_{4j}^U \leqslant 1$, $0 \leqslant h_j^L \leqslant h_j^U \leqslant 1$, $w_{1j}^U \leqslant w_{1j}^L$, and $w_{4j}^L \leqslant w_{4j}^U$. An IT2TrFN W, which is defined on the universe of discourse X, is an object of the following form:

$$W = \left\{\left\langle x_j, \left[W_j^L, W_j^U\right]\right\rangle | x_j \in X,\ j = 1, 2, \ldots, n\right\}, \quad (5)$$

where $W_j^L = \left(w_{1j}^L, w_{2j}^L, w_{3j}^L, w_{4j}^L; h_j^L\right)$, $W_j^U = \left(w_{1j}^U, w_{2j}^U, w_{3j}^U, w_{4j}^U; h_j^U\right)$, and $W_j^L \subset W_j^U$.

3. Extended QUALIFLEX method with IT2TrFNs

In the decision environment of IT2TrFNs, the extended QUALIFLEX method was designed to obtain the best order of the alternatives based on the level of concordance and to select the most preferred alternative from the set of alternatives. The proposed method uses successive permutations of all possible rankings of alternatives and recognizes the concordance/discordance index for all permutation rankings of the alternatives using a signed distance-based method.

3.1. Signed distance-based approach

The concept of signed distances, also referred to as oriented distances or directed distances, can be used to study rankings of fuzzy numbers (Chiang, 2001; Chen and Ouyang, 2006). Despite

Table 1
Linguistic variables and their corresponding IT2TrFNs.

Importance	Rating	Corresponding IT2TrFNs
Absolutely low (AL)	Absolutely poor (AP)	[(0.0, 0.0, 0.0, 0.0; 1.0), (0.0, 0.0, 0.0, 0.0; 1.0)]
Very low (VL)	Very poor (VP)	[(0.0075, 0.0075, 0.015, 0.0525; 0.8), (0.0, 0.0, 0.02, 0.07; 1.0)]
Low (L)	Poor (P)	[(0.0875, 0.12, 0.16, 0.1825; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]
Medium low (ML)	Medium poor (MP)	[(0.2325, 0.255, 0.325, 0.3575; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
Medium (M)	Fair (F)	[(0.4025, 0.4525, 0.5375, 0.5675; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]
Medium high (MH)	Medium good (MG)	[(0.65, 0.6725, 0.7575, 0.79; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
High (H)	Good (G)	[(0.7825, 0.815, 0.885, 0.9075; 0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]
Very high (VH)	Very good (VG)	[(0.9475, 0.985, 0.9925, 0.9925; 0.8), (0.93, 0.98, 1.0, 1.0; 1.0)]
Absolutely high (AH)	Absolutely good (AG)	[(1.0, 1.0, 1.0, 1.0; 1.0), (1.0, 1.0, 1.0, 1.0; 1.0)]

the multiple ranking methods, no decision-maker is able to consistently rank fuzzy numbers using human intuition in all cases (Abbasbandy and Asady, 2006). Certain limitations were found when ranking fuzzy numbers with the following methods: the coefficient of variation (i.e., CV index), the distance between fuzzy sets, the centroid point and the original point, and the weighted mean value (Yao and Wu, 2000; Abbasbandy and Asady, 2006). The signed distance method is able to effectively rank various fuzzy numbers and their images (Yao and Wu, 2000). In addition, the signed distance method calculations are less complicated than the calculations from other approaches (Abbasbandy and Asady, 2006). The signed distance method can also use both positive and negative values to define the ordering of fuzzy numbers. This method is different from ordinary distance measurement techniques (Yao and Wu, 2000), and this study employs a signed distance-based approach to compare the IT2TrFN values.

Consider a decision matrix D that refers to m alternatives on n criteria. The IT2TrFN rating A_{ij} was expressed as $A_{ij}=\left[A_{ij}^L,A_{ij}^U\right]=\left[\left(a_{1ij}^L,a_{2ij}^L,a_{3ij}^L,a_{4ij}^L;h_{ij}^L\right),\left(a_{1ij}^U,a_{2ij}^U,a_{3ij}^U,a_{4ij}^U;h_{ij}^U\right)\right]$. Let the level 1 fuzzy number $\tilde{0}_1$ map onto the vertical axis at the origin. Following the discussions of Chen (2011a,b), assume that $h_{ij}^L\neq 0$, and $0<h_{ij}^L\leqslant h_{ij}^U\leqslant 1$. The signed distance from A_{ij} to $\tilde{0}_1$ is as follows:

$$d(A_{ij},\tilde{0}_1)=\frac{1}{8}\left(a_{1ij}^L+a_{2ij}^L+a_{3ij}^L+a_{4ij}^L+4a_{1ij}^U+2a_{2ij}^U+2a_{3ij}^U+4a_{4ij}^U+3\left(a_{2ij}^U+a_{3ij}^U-a_{1ij}^U-a_{4ij}^U\right)\frac{h_{ij}^L}{h_{ij}^U}\right). \quad (6)$$

Let A_{ij} and $A_{i'j'}$ be two IT2TrFN ratings. Because the signed distances $d(A_{ij},\tilde{0}_1)$ and $d(A_{i'j'},\tilde{0}_1)$ are real numbers, they satisfy linear ordering. In other words, one of the following three conditions must hold: $d(A_{ij},\tilde{0}_1)>d(A_{i'j'},\tilde{0}_1)$, $d(A_{ij},\tilde{0}_1)=d(A_{i'j'},\tilde{0}_1)$, or $d(A_{ij},\tilde{0}_1)<d(A_{i'j'},\tilde{0}_1)$. It follows that the signed distance based on IT2TrFNs satisfies the law of trichotomy. A comparison of the IT2TrFN ratings can be drawn via the signed distance from the IT2TrFN to $\tilde{0}_1$.

3.2. The extended QUALIFLEX method

This paper proposes a signed distance-based approach to identify the concordance/discordance index. Assume that the alternative A_ρ is ranked higher than or equal to A_β. Given the alternative set A with m alternatives, $m!$ permutations of the ranking of the alternatives exist. Let P_l denote the lth permutation:

$$P_l=(\ldots,A_\rho,\ldots,A_\beta,\ldots),\quad \text{for } l=1,2,\ldots,m!. \quad (7)$$

The evaluation values of A_ρ and A_β with respect to each criterion $x_j\in X$ are

$$A_{\rho j}=\left[A_{\rho j}^L,A_{\rho j}^U\right]=\left[\left(a_{1\rho j}^L,a_{2\rho j}^L,a_{3\rho j}^L,a_{4\rho j}^L;h_{\rho j}^L\right),\left(a_{1\rho j}^U,a_{2\rho j}^U,a_{3\rho j}^U,a_{4\rho j}^U;h_{\rho j}^U\right)\right]$$

and

$$A_{\beta j}=\left[A_{\beta j}^L,A_{\beta j}^U\right]=\left[\left(a_{1\beta j}^L,a_{2\beta j}^L,a_{3\beta j}^L,a_{4\beta j}^L;h_{\beta j}^L\right),\left(a_{1\beta j}^U,a_{2\beta j}^U,a_{3\beta j}^U,a_{4\beta j}^U;h_{\beta j}^U\right)\right]$$

The signed distances of $d(A_{\rho j},\tilde{0}_1)$ and $d(A_{\beta j},\tilde{0}_1)$ can be used to order $A_{\rho j}$ and $A_{\beta j}$. Examples of typical relationships between $A_{\rho j}$ and $A_{\beta j}$ are shown in Figs. 1–3. Figs. 1 and 2 indicate situations with no discordance. If x_j is the only criterion to be considered, $A_{\rho j}$ ranks above $A_{\beta j}$ in Fig. 1 (because $d(A_{\rho j},\tilde{0}_1)>d(A_{\beta j},\tilde{0}_1)$); moreover, $A_{\rho j}$ and $A_{\beta j}$ is indifferent in Fig. 2 (because $d(A_{\rho j},\tilde{0}_1)=d(A_{\beta j},\tilde{0}_1)$). Thus, there is concordance between the signed distance-based ranking orders and the preorders of $A_{\rho j}$ and $A_{\beta j}$; while there is ex aequo if $A_{\rho j}$ and $A_{\beta j}$ have the same rank in the signed distance-based ranking. Fig. 3 shows a conflict between the signed distance-based ranking orders and the preorders of $A_{\rho j}$ and $A_{\beta j}$. We observe that $A_{\rho j}$ ranks below $A_{\beta j}$ because $d(A_{\rho j},\tilde{0}_1)<d(A_{\beta j},\tilde{0}_1)$, but this is discordant with the preorders. Following the discussion above, the difference between $d(A_{\rho j},\tilde{0}_1)$ and $d(A_{\beta j},\tilde{0}_1)$ can be employed to measure the levels of concordance or discordance through a concordance/discordance index.

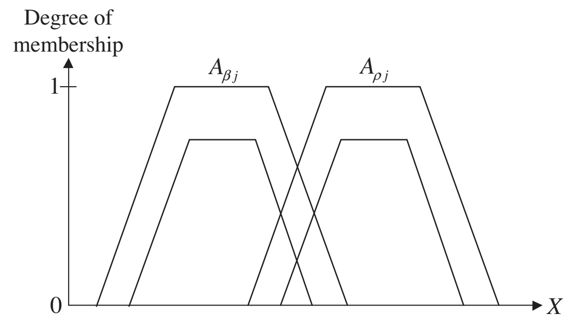

Fig. 1. Concordance with the preorders.

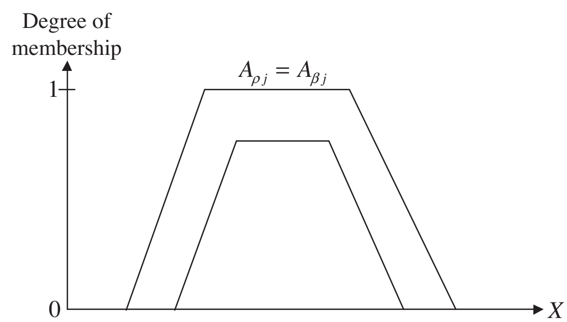

Fig. 2. Ex aequo with the preorders.

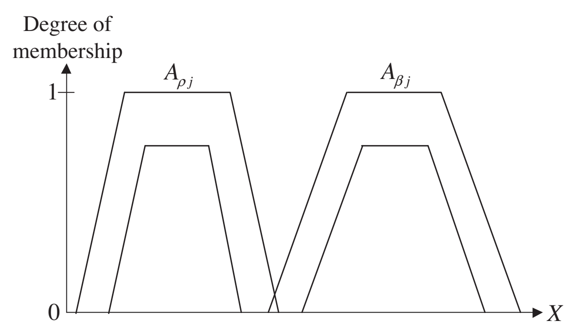

Fig. 3. Discordance with the preorders.

The concordance/discordance index, $I_j^l(A_\rho,A_\beta)$, for each pair of alternatives, (A_ρ, A_β), $A_\rho, A_\beta\in A$, at the level of preorder, according to the criterion $x_j\in X$ and the ranking corresponding to the lth permutation, is as follows:

$$I_j^l(A_\rho,A_\beta)=d(A_{\rho j},\tilde{0}_1)-d(A_{\beta j},\tilde{0}_1). \quad (8)$$

There are concordance, ex aequo, and discordance if $I_j^l(A_\rho,A_\beta)>0$, $I_j^l(A_\rho,A_\beta)=0$, and $I_j^l(A_\rho,A_\beta)<0$, respectively. Furthermore, the concordance/discordance index I_j^l, between the preorder according to the criterion x_j and the ranking corresponding to the lth permutation, is:

$$I_j^l = \sum_{A_\rho,A_\beta \in A} I_j^l(A_\rho, A_\beta) = \sum_{A_\rho,A_\beta \in A} (d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)). \quad (9)$$

The index $I_j^l(A_\rho, A_\beta)$ can be considered an evaluation value of the pair of alternatives (A_ρ, A_β) in the lth permutation according to the criterion x_j. No objection exists to assigning unequal importance to each criterion for practical applications. Considering the importance weight W_j of each criterion $x_j \in X$, the weighted concordance/discordance index $I^l(A_\rho, A_\beta)$ for each pair of alternatives (A_ρ, A_β) $(A_\rho, A_\beta \in A)$ at the level of preorder with respect to the n criteria in X and the ranking corresponding to the lth permutation is

$$I^l(A_\rho, A_\beta) = \sum_{j=1}^{n} I_j^l(A_\rho, A_\beta) \cdot W_j = \sum_{j=1}^{n} (d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)) \cdot W_j. \quad (10)$$

Combining I_j^l and $I^l(A_\rho, A_\beta)$, the comprehensive concordance/discordance index I^l for the lth permutation is

$$I^l = \sum_{A_\rho,A_\beta \in A} \sum_{j=1}^{n} I_j^l(A_\rho, A_\beta) \cdot W_j = \sum_{A_\rho,A_\beta \in A} \sum_{j=1}^{n} (d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)) \cdot W_j. \quad (11)$$

The evaluation criterion of the chosen hypothesis for ranking of the alternatives is the arithmetic sum of all weighted differences of signed distances corresponding to the element-by-element consistency. The signed distance $d(I^l, \tilde{0}_1)$ is computed to compare the comprehensive concordance/discordance index for all permutations:

$$d(I^l, \tilde{0}_1) = d\left(\sum_{A_\rho,A_\beta \in A} \sum_{j=1}^{n} I_j^l(A_\rho, A_\beta) \cdot W_j, \tilde{0}_1\right) = d\left(\sum_{A_\rho,A_\beta \in A} \sum_{j=1}^{n} (d(A_{\rho j}, \tilde{0}_1) - d(A_{\beta j}, \tilde{0}_1)) \cdot W_j, \tilde{0}_1\right), \text{ for } l = 1, 2, \ldots, m!. \quad (12)$$

For each permutation P_l (l = 1, 2, …, m!), we choose the maximum $d(I^l, \tilde{0}_1)$ value among all comprehensive concordance/discordance indices I^l, and the optimal ranking order of the alternatives can be obtained correspondingly.

3.3. The proposed algorithm

Based on IT2TrFNs, the extended QUALIFLEX method for solving a multiple criteria decision-making problem can be summarized in the following steps:

Step 1: Formulate a multiple criteria decision-making problem. Specify the evaluation criteria ($X = \{x_1, x_2, \ldots, x_n\}$) and generate feasible alternatives ($A = \{A_1, A_2, \ldots, A_m\}$).

Step 2: Select appropriate linguistic variables and translation standards (e.g., the rating scale in Table 1) for conversion into IT2TrFNs for the importance weights of criteria and the linguistic ratings for the alternatives with respect to each criterion.

Step 3: Investigate the decision-maker to provide appropriate linguistic weighting and rating terms that best represent the importance of the criteria and the alternative evaluation of each criterion, respectively.

Step 4: Convert the linguistic evaluations into IT2TrFNs to obtain the rating A_{ij} of the alternative A_i on the criterion x_j and the importance weight W_j of the criterion x_j. Next, construct the decision matrix D in (2) and the criterion importance W in (5).

Step 5: Calculate the signed distance $d(A_{ij}, \tilde{0}_1)$ for each A_{ij} in D using (6).

Step 6: List all of the possible m! permutations of the m alternatives that must be tested. Let P_l ($l = 1, 2, \ldots, m!$) denote the lth permutation using (7).

Step 7: Compute the concordance/discordance index $I_j^l(A_\rho, A_\beta)$ for each pair of (A_ρ, A_β) in the permutation P_l with respect to the criterion $x_j \in X$ using (8).

Step 8: Consider the importance weight W_j of each criterion to compute the $I_j^l(A_\rho, A_\beta) \cdot W_j$ values. Next, apply (10) to determine the weighted concordance/discordance index $I^l(A_\rho, A_\beta)$ for each pair of (A_ρ, A_β) in P_l.

Step 9: Derive the comprehensive concordance/discordance index I^l for each P_l using (11). Apply (12) to derive the signed distance $d(I^l, \tilde{0}_1)$ for each permutation P_l. The permutation with the maximal signed distance value is the optimal ranking order of the alternatives.

4. A case study for medical decision making

The following practical example involves a medical decision-making problem concerning acute inflammatory demyelinating disease. The example demonstrates the effective use of the extended QUALIFLEX method within an IT2TrFN framework.

4.1. Decision context

The case comes from the Department of Neurology, Chang Gung Memorial Hospital in Taiwan. The patient was a 48-year-old widowed female with a history of diabetes mellitus. Her physician made a diagnosis of acute inflammatory demyelinating disease. Acute inflammatory demyelinating polyneuropathy is a disorder affecting the peripheral nervous system. Ascending paralysis, manifesting as weakness beginning in the feet and hands and migrating toward the trunk, is the most typical symptom. The disease can cause life-threatening complications, particularly if the breathing muscles are affected or if there is a dysfunction of the autonomic nervous system. The disease is usually triggered by an acute infection. Recovery usually begins after the fourth week from the onset of the disorder. Approximately 80% of patients completely recover within a few months to a year, although minor symptoms may persist, such as areflexia. Approximately 5–10% of patients recover with severe disability, primarily involving severe proximal motor and sensory axonal damage and an inability to regenerate axons.

The attending physician assessed the patient's medical history and her current physical conditions and provided three treatment options, including steroid therapy (A_1), plasmapheresis (A_2), and albumin immune therapy (A_3). To assist the patient and her family's understanding of the advantages and disadvantages of each treatment, the physician provided information based on several evaluative criteria, including survival rate (x_1), severity of the side effects (x_2), severity of the complications (x_3), probability of a cure (x_4), discomfort index of the treatment (x_5), cost (x_6), number of days of hospitalization (x_7), probability of a recurrence (x_8), and self-care capacity (x_9). Additionally, the physician described three treatment methods using these criteria, as summarized in Table 2. The physician wanted the patient and her family members to discuss and assess the treatment options thoroughly.

4.2. Illustration of the proposed algorithm

In Step 1, we have $X = \{x_1, x_2, \ldots, x_9\}$ and $A = \{A_1, A_2, A_3\}$. In Step 2, the nine-point linguistic variables and translation standards from

Table 2
Descriptions of the treatment methods using the evaluative criteria.

Steroid therapy (A_1)
(1) A high survival rate
(2) There are no obvious side effects
(3) The possibility of a sepsis as a complication
(4) About a 40% probability of a cure
(5) No pain/discomfort during treatment
(6) Health insurance covers most of the expenses, with a low out-of-pocket expense
(7) A very long hospitalization
(8) A significantly high probability of a recurrence
(9) A poor prognosis for the patient's self-care ability

Plasmapheresis (A_2)
(1) A very high survival rate
(2) The possibility of a blood pressure drop
(3) The low possibility of shock as a complication
(4) A very high probability of a cure
(5) A dialysis is used during the treatment, which generates more discomfort than the other treatments
(6) Health insurance covers some of the expenses, with a moderately higher out-of-pocket expense
(7) A moderate hospitalization
(8) A low probability of a recurrence
(9) A moderate prognosis for the patient's self-care capacity

Albumin immune therapy (A_3)
(1) A very high survival rate
(2) The possibility of a cold or weariness
(3) The possibility of a sepsis as a complication
(4) A high probability of a cure
(5) No pain/discomfort during treatment
(6) Low coverage by the patient's health insurance and very high out-of-pocket expenses (about NT$100K)
(7) A slightly shorter hospitalization than A_2
(8) A low probability of a recurrence
(9) A moderate prognosis for the patient's self-care capacity

Table 1 were selected for conversion into IT2TrFNs. In Step 3, the criterion importance weights were assessed, and the three treatment methods were evaluated based on the nine criteria using the linguistic rating system. The ratings of the criterion importance and the treatment options on the criteria were completed by the patient and her family members. In general, human decision-making behavior is always subjective to a certain extent. Decision-makers act and react based on their perceptions and not based on objective reality. For each decision-maker, reality is a completely personal phenomenon based on individual needs, wants, personality traits, values, experiences, and subjective judgments. Because individuals make decisions and perform actions according to what they perceive to be reality, it is important to take human subjectivity into account as part of the decision-making process. By means of the linguistic rating system, the researcher was able to collect the opinions of the patient and her family members easily regarding the criterion importance and the ratings of the three treatment options. The investigation results are presented in Table 3.

In Step 4, these linguistic evaluations were converted into IT2TrFN values. Then, the criterion importance W and the decision matrix D were subsequently obtained. In Step 5, we computed the signed distance $d(A_{ij}, \tilde{0}_1)$ for each A_{ij} in D. In Step 6, there are 6 (=3!) permutations of the rankings for each of the alternatives that must be tested: $P_1 = (A_1, A_2, A_3)$, $P_2 = (A_1, A_3, A_2)$, $P_3 = (A_2, A_1, A_3)$, $P_4 = (A_2, A_3, A_1)$, $P_5 = (A_3, A_1, A_2)$, and $P_6 = (A_3, A_2, A_1)$. In Step 7, for each pair of alternatives (A_ρ, A_β) in the permutation P_l with respect to each criterion x_j, the results of the concordance/discordance index $I_j^l(A_\rho, A_\beta)$ are presented in Table 4.

In Step 8, we computed the values of $I_j^l(A_\rho, A_\beta) \cdot W_j$ and $I^l(A_\rho, A_\beta)$ for each pair of (A_ρ, A_β) in P_l. Considering the first permutation P_1, for example, the results of P_1 are indicated in Table 5.

Table 3
The criterion importance weights and therapeutic ratings.

Criteria	Importance weights	Treatment options		
		A_1	A_2	A_3
x_1 (survival rate)	AH	G	VG	VG
x_2 (severity of the side effects)	L	G	F	MG
x_3 (severity of the complications)	ML	P	MG	P
x_4 (probability of a cure)	AH	MP	VG	G
x_5 (discomfort index of the treatment)	VL	VG	P	VG
x_6 (cost)	M	G	MP	AP
x_7 (number of days of hospitalization)	VH	VP	MP	F
x_8 (probability of a recurrence)	H	AP	G	G
x_9 (self-care capacity)	MH	P	F	F

Table 4
The results of the concordance/discordance index.

P_1	$I_j^1(A_1,A_2)$	$I_j^1(A_1,A_3)$	$I_j^1(A_2,A_3)$	P_2	$I_j^2(A_1,A_3)$	$I_j^2(A_1,A_2)$	$I_j^2(A_3,A_2)$
x_1	−0.2679	−0.2679	0.0000	x_1	−0.2679	−0.2679	0.0000
x_2	0.7133	0.2635	−0.4498	x_2	0.2635	0.7133	0.4498
x_3	−1.1565	0.0000	1.1565	x_3	0.0000	−1.1565	−1.1565
x_4	−1.3814	−1.1135	0.2679	x_4	−1.1135	−1.3814	−0.2679
x_5	1.6879	0.0000	−1.6879	x_5	0.0000	1.6879	1.6879
x_6	1.1135	1.6968	0.5833	x_6	1.6968	1.1135	−0.5833
x_7	−0.5480	−0.9482	−0.4002	x_7	−0.9482	−0.5480	0.4002
x_8	−1.6968	−1.6968	0.0000	x_8	−1.6968	−1.6968	0.0000
x_9	−0.7067	−0.7067	0.0000	x_9	−0.7067	−0.7067	0.0000
P_3	$I_j^3(A_2,A_1)$	$I_j^3(A_2,A_3)$	$I_j^3(A_1,A_3)$	P_4	$I_j^4(A_2,A_3)$	$I_j^4(A_2,A_1)$	$I_j^4(A_3,A_1)$
x_1	0.2679	0.0000	−0.2679	x_1	0.0000	0.2679	0.2679
x_2	−0.7133	−0.4498	0.2635	x_2	−0.4498	−0.7133	−0.2635
x_3	1.1565	1.1565	0.0000	x_3	1.1565	1.1565	0.0000
x_4	1.3814	0.2679	−1.1135	x_4	0.2679	1.3814	1.1135
x_5	−1.6879	−1.6879	0.0000	x_5	−1.6879	−1.6879	0.0000
x_6	−1.1135	0.5833	1.6968	x_6	0.5833	−1.1135	−1.6968
x_7	0.5480	−0.4002	−0.9482	x_7	−0.4002	0.5480	0.9482
x_8	1.6968	0.0000	−1.6968	x_8	0.0000	1.6968	1.6968
x_9	0.7067	0.0000	−0.7067	x_9	0.0000	0.7067	0.7067
P_5	$I_j^5(A_3,A_1)$	$I_j^5(A_3,A_2)$	$I_j^5(A_1,A_2)$	P_6	$I_j^6(A_3,A_2)$	$I_j^6(A_3,A_1)$	$I_j^6(A_2,A_1)$
x_1	0.2679	0.0000	−0.2679	x_1	0.0000	0.2679	0.2679
x_2	−0.2635	0.4498	0.7133	x_2	0.4498	−0.2635	−0.7133
x_3	0.0000	−1.1565	−1.1565	x_3	−1.1565	0.0000	1.1565
x_4	1.1135	−0.2679	−1.3814	x_4	−0.2679	1.1135	1.3814
x_5	0.0000	1.6879	1.6879	x_5	1.6879	0.0000	−1.6879
x_6	−1.6968	−0.5833	1.1135	x_6	−0.5833	−1.6968	−1.1135
x_7	0.9482	0.4002	−0.5480	x_7	0.4002	0.9482	0.5480
x_8	1.6968	0.0000	−1.6968	x_8	0.0000	1.6968	1.6968
x_9	0.7067	0.0000	−0.7067	x_9	0.0000	0.7067	0.7067

In Step 9, the comprehensive concordance/discordance index I^l was calculated for each P_l, as follows:

$$I^1 = [(-7.6925, -7.2317, -6.3271, -5.8485; 0.8), (-8.5522, -7.6996, -5.8639, -4.9698; 1)],$$

$$I^2 = [(-8.4773, -8.0604, -7.1558, -6.6334; 0.8), (-9.3242, -8.5315, -6.6959, -5.7418; 1)],$$

$$I^3 = [(-0.1371, 0.3764, 1.2811, 1.7068; 0.8), (-1.0192, -0.0859, 1.7498, 2.5632; 1)],$$

$$I^4 = [(6.6334, 7.1558, 8.0604, 8.4773; 0.8), (5.7418, 6.6959, 8.5315, 9.3242; 1)],$$

$$I^5 = [(-1.7068, -1.2811, -0.3764, 0.1371; 0.8), (-2.5632, -1.7498, 0.0859, 1.0192; 1)],$$

Table 5
Sample results of the weighted concordance/discordance index (for P_1).

Criteria	$I_j^1(A_1, A_2) \cdot W_j$
x_1	[(−0.2679, −0.2679, −0.2679, −0.2679; 1), (−0.2679, −0.2679, −0.2679, −0.2679; 1)]
x_2	[(0.0624, 0.0856, 0.1141, 0.1302; 0.8), (0.0285, 0.0713, 0.1284, 0.1641; 1)]
x_3	[(−0.4134, −0.3759, −0.2949, −0.2689; 0.8), (−0.4857, −0.4163, −0.2544, −0.1966; 1)]
x_4	[(−1.3814, −1.3814, −1.3814, −1.3814; 1), (−1.3814, −1.3814, −1.3814, −1.3814; 1)]
x_5	[(0.0127, 0.0127, 0.0253, 0.0886; 0.8), (0.0000, 0.0000, 0.0338, 0.1182; 1)]
x_6	[(0.4482, 0.5039, 0.5985, 0.6319; 0.8), (0.3563, 0.4565, 0.6458, 0.7238; 1)]
x_7	[(−0.5439, −0.5439, −0.5398, −0.5192; 0.8), (−0.5480, −0.5480, −0.5370, −0.5096; 1)]
x_8	[(−1.5398, −1.5017, −1.3829, −1.3277; 0.8), (−1.6459, −1.5611, −1.3235, −1.2217; 1)]
x_9	[(−0.5583, −0.5353, −0.4753, −0.4594; 0.8), (−0.6078, −0.5654, −0.4452, −0.4099; 1)]
$I^1(A_1, A_2)$ = [(−4.1815, −4.0039, −3.6042, −3.3738; 0.8), (−4.5518, −4.2122, −3.4015, −2.9811; 1)]	
	$I_j^1(A_1, A_3) \cdot W_j$
x_1	[(−0.2679, −0.2679, −0.2679, −0.2679; 1), (−0.2679, −0.2679, −0.2679, −0.2679; 1)]
x_2	[(0.0231, 0.0316, 0.0422, 0.0481; 0.8), (0.0105, 0.0264, 0.0474, 0.0606; 1)]
x_3	[(0.0000, 0.0000, 0.0000, 0.0000; 0.8), (0.0000, 0.0000, 0.0000, 0.0000; 1)]
x_4	[(−1.1135, −1.1135, −1.1135, −1.1135; 1), (−1.1135, −1.1135, −1.1135, −1.1135; 1)]
x_5	[(0.0000, 0.0000, 0.0000, 0.0000; 0.8), (0.0000, 0.0000, 0.0000, 0.0000; 1)]
x_6	[(0.6830, 0.7678, 0.9120, 0.9629; 0.8), (0.5430, 0.6957, 0.9841, 1.1029; 1)]
x_7	[(−0.9411, −0.9411, −0.9340, −0.8984; 0.8), (−0.9482, −0.9482, −0.9292, −0.8818; 1)]
x_8	[(−1.5398, −1.5017, −1.3829, −1.3277; 0.8), (−1.6459, −1.5611, −1.3235, −1.2217; 1)]
x_9	[(−0.5583, −0.5353, −0.4753, −0.4594; 0.8), (−0.6078, −0.5654, −0.4452, −0.4099; 1)]
$I^1(A_1, A_3)$ = [(−3.7146, −3.5601, −3.2193, −3.0559; 0.8), (−4.0297, −3.7340, −3.0478, −2.7313; 1)]	
	$I_j^1(A_2, A_3) \cdot W_j$
x_1	[(0.0000, 0.0000, 0.0000, 0.0000; 1), (0.0000, 0.0000, 0.0000, 0.0000; 1)]
x_2	[(−0.0821, −0.0720, −0.0540, −0.0394; 0.8), (−0.1035, −0.0810, −0.0450, −0.0180; 1)]
x_3	[(0.2689, 0.2949, 0.3759, 0.4134; 0.8), (0.1966, 0.2544, 0.4163, 0.4857; 1)]
x_4	[(0.2679, 0.2679, 0.2679, 0.2679; 1), (0.2679, 0.2679, 0.2679, 0.2679; 1)]
x_5	[(−0.0886, −0.0253, −0.0127, −0.0127; 0.8), (−0.1182, −0.0338, 0.0000, 0.0000; 1)]
x_6	[(0.2348, 0.2639, 0.3135, 0.3310; 0.8), (0.1867, 0.2392, 0.3383, 0.3791; 1)]
x_7	[(−0.3972, −0.3972, −0.3942, −0.3792; 0.8), (−0.4002, −0.4002, −0.3922, −0.3722; 1)]
x_8	[(0.0000, 0.0000, 0.0000, 0.0000; 0.8), (0.0000, 0.0000, 0.0000, 0.0000; 1)]
x_9	[(0.0000, 0.0000, 0.0000, 0.0000; 0.8), (0.0000, 0.0000, 0.0000, 0.0000; 1)]
$I^1(A_2, A_3)$ = [(0.2037, 0.3323, 0.4965, 0.5812; 0.8), (0.0294, 0.2466, 0.5854, 0.7426; 1)]	

$$I^6 = [(5.8485, 6.3271, 7.2317, 7.6925; 0.8), (4.9698, 5.8639, 7.6996, 8.5522; 1)].$$

Furthermore, the signed distance $d(I^l, \tilde{0}_1)$ was computed for each permutation P_l as follows: $d(I^1, \tilde{0}_1) = -13.5518$, $d(I^2, \tilde{0}_1) = -15.1791$, $d(I^3, \tilde{0}_1) = 1.6273$, $d(I^4, \tilde{0}_1) = 15.1791$, $d(I^5, \tilde{0}_1) = -1.6273$, and $d(I^6, \tilde{0}_1) = 13.5518$, where $d(I^4, \tilde{0}_1)$ gives the maximal value. The best permutation is $P_4 = (A_2, A_3, A_1)$, and thus the best order of the candidate treatment options is $A_2 \succ A_3 \succ A_1$. Therefore, the best choice for the patient is plasmapheresis (A_2). Ultimately, the patient and her family decided to adopt plasmapheresis as the treatment of choice. The patient is currently undergoing post-treatment rehabilitation.

Given the increasing awareness of health rights, the rights of patients have drawn increasing attention. Therefore, the focus of providing healthcare has shifted from the sole perspective of the medical personnel to a patient-centered approach. Diseases are sources of stress for patients and their family members. When a patient is subjected to an emergent and life-threatening disease, selecting the most appropriate treatment is a difficult and complex process. This process involves highly complex decision making by the patients and their family members. The physician usually provides a limited number of treatment protocols for a patient to select from. The protocols are limited because patients and/or their family members may find it difficult to choose between multiple treatments that have equally significant therapeutic effects. The medical decision-making process involves numerous complex and possibly contradictory assessment criteria. The proposed extended QUALIFLEX method with IT2TrFNs is useful for handling complicated patient-centered decision-making problems that involve comprehensive criteria and limited alternatives.

4.3. Discussions of computational costs and applications

The proposed extended QUALIFLEX method does not require complicated computations in the implementation procedure for each permutation P_l, as demonstrated in the illustration of selecting a suitable treatment method. The degree of computational complexity corresponding to each P_l is quite low. However, the number of permutations rapidly increases with an increasing number of alternatives. For example, 3,628,800 (=10!) permutations of the ranking of the alternatives exist if $m = 10$. Therefore, an evident limitation of the proposed extended QUALIFLEX method with IT2TrFNs relative to the existing methods is the tedious computations required for handling a multiple criteria decision-making problem with a sufficiently large number of alternatives.

However, in numerous practical decision-making problems, the decision-maker may be unconcerned about this disadvantage. Using a practical example from clinical medicine at Chang Gung Memorial Hospital in Taiwan, this paper illustrates the feasibility and effectiveness of the extended QUALIFLEX method in the case of limited alternatives. Although the number of permutations increases drastically with an increase in the number of alternatives, the arithmetic computations corresponding to each P_l are still easy to implement. In other words, the number of permutations has little influence upon the individual computational complexity for each P_l. Considering that a decision-making problem refers to many alternatives, the difficulty of implementing the arithmetic computations for all permutations can be significantly overcome with the help of powerful computer hardware. Nevertheless, to avoid troublesome computations, it is cautiously suggested that the proposed method may be not applicable to decision-making problems with large numbers of alternatives. The proposed method is also suitable for situations where the number of criteria markedly exceeds the number of alternatives. Examples of such problems include decisions in public or government policies, natural resources management, high-risk decision activities, decision-making problems with high involvement, and other complex decisions using many criteria to evaluate a limited number of alternatives.

4.4. Comparative analysis and discussions

A comparative study was conducted to validate the results of the proposed extended QUALIFLEX method with those from another approach. We based the analysis on the same medical decision-making problem and chose a well-known and widely used outranking method, the ELECTRE approach, to facilitate the comparative analysis. By expanding the interval-valued fuzzy ELECTRE methods (Vahdani et al., 2010; Vahdani and Hadipour, 2011), we propose the modified ELECTRE method to handle the IT2TrFN data appropriately and apply it to the same medical decision-making problem of acute inflammatory demyelinating disease.

The weighted evaluation $W_j \otimes A_{ij}$ of the alternative A_i on the criterion x_j was calculated in the following way:

$$W_j \otimes A_{ij} = \left[\left(w_{1j}^L \times a_{1ij}^L, w_{2j}^L \times a_{2ij}^L, w_{3j}^L \times a_{3ij}^L, w_{4j}^L \times a_{4ij}^L; \min\left(h_j^L, h_{ij}^L\right)\right), \left(w_{1j}^U \times a_{1ij}^U, w_{2j}^U \times a_{2ij}^U, w_{3j}^U \times a_{3ij}^U, w_{4j}^U \times a_{4ij}^U; \min\left(h_j^U, h_{ij}^U\right)\right)\right] = \left[\left(a_{1ij}^{WL}, a_{2ij}^{WL}, a_{3ij}^{WL}, a_{4ij}^{WL}; h_{ij}^{WL}\right), \left(a_{1ij}^{WU}, a_{2ij}^{WU}, a_{3ij}^{WU}, a_{4ij}^{WU}; h_{ij}^{WU}\right)\right]. \quad (13)$$

Next, we calculate the signed distance $d(W_j \otimes A_{ij}, \tilde{0}_1)$, as shown in Table 6.

The concordance set $\mathbb{C}_{\rho\beta}$ of the pair of (A_ρ, A_β) is composed of all criteria for which A_ρ is preferred to A_β by comparing their signed distances. That is,

$$\mathbb{C}_{\rho\beta} = \{j | d(W_j \otimes A_{\rho j}, \tilde{0}_1) \geqslant d(W_j \otimes A_{\beta j}, \tilde{0}_1)\}. \quad (14)$$

In contrast, the complementary part is called the discordance set $\mathbb{N}_{\rho\beta}$, which is

$$\mathbb{N}_{\rho\beta} = \{j | d(W_j \otimes A_{\rho j}, \tilde{0}_1) < d(W_j \otimes A_{\beta j}, \tilde{0}_1)\}. \quad (15)$$

According to the signed distances in Table 6, we obtain $\mathbb{C}_{\rho\beta}$ and $\mathbb{N}_{\rho\beta}$ as follows:

$\mathbb{C}_{12} = \{2,5,6\}$, $\mathbb{C}_{13} = \{2,3,5,6\}$, $\mathbb{C}_{21} = \{1,3,4,7,8,9\}$, $\mathbb{C}_{23} = \{1,3,4,6,8,9\}$,

$\mathbb{C}_{31} = \{1,3,4,5,7,8,9\}$, $\mathbb{C}_{32} = \{1,2,5,7,8,9\}$, $\mathbb{N}_{12} = \{1,3,4,7,8,9\}$, $\mathbb{N}_{13} = \{1,4,7,8,9\}$, $\mathbb{N}_{21} = \{2,5,6\}$, $\mathbb{N}_{23} = \{2,5,7\}$, $\mathbb{N}_{31} = \{2,6\}$, and $\mathbb{N}_{32} = \{3,4,6\}$.

Table 6
The results of the signed distance from $W_j \otimes A_{ij}$ to $\tilde{0}_1$.

A_{ij}	$d(W_j \otimes A_{ij}, \tilde{0}_1)$	A_{ij}	$d(W_j \otimes A_{ij}, \tilde{0}_1)$	A_{ij}	$d(W_j \otimes A_{ij}, \tilde{0}_1)$
A_{11}	1.6968	A_{21}	1.9647	A_{31}	1.9647
A_{12}	0.2435	A_{22}	0.1473	A_{32}	0.2084
A_{13}	0.0894	A_{23}	0.4330	A_{33}	0.0894
A_{14}	0.5833	A_{24}	1.9647	A_{34}	1.6968
A_{15}	0.0352	A_{25}	0.0069	A_{35}	0.0352
A_{16}	0.8509	A_{26}	0.3032	A_{36}	0.0000
A_{17}	0.0352	A_{27}	0.5758	A_{37}	0.9700
A_{18}	0.0000	A_{28}	1.4524	A_{38}	1.4524
A_{19}	0.2084	A_{29}	0.7239	A_{39}	0.7239

The relative value of the concordance set is measured by means of the concordance index. The concordance index is proportional to the sum of the IT2TrFN weights associated with those criteria that are contained in the concordance set. We denote $W_1 \oplus W_2 \oplus \cdots \oplus W_n = \oplus_{j=1}^n W_j$. The concordance index $\mathcal{C}_{\rho\beta}$ for the pair of (A_ρ, A_β) is defined as follows:

$$\mathcal{C}_{\rho\beta} = (\oplus_{j \in \mathbb{C}_{\rho\beta}} W_j) \emptyset \left(\oplus_{j=1}^n W_j\right). \quad (16)$$

The concordance index reflects the relative dominance of A_ρ over A_β based on the relative importance attached to the successive decision criteria. The threshold IT2TrFN value for $\mathcal{C}_{\rho\beta}$ is designated as the average concordance index $\overline{\mathcal{C}}$; that is,

$$\overline{\mathcal{C}} = \left(\oplus_{\rho=1,\rho\neq\beta}^m \oplus_{\beta=1,\beta\neq\rho}^m \mathcal{C}_{\rho\beta}\right) \Big/ m(m-1). \quad (17)$$

In the medical decision problem, the concordance indices are derived as follows:

$\mathcal{C}_{12} = [(0.0850, 0.1022, 0.1342, 0.1570; 0.8), (0.0581, 0.0870, 0.1523, 0.1996; 1)]$,

$\mathcal{C}_{13} = [(0.1248, 0.1472, 0.1955, 0.2270; 0.8), (0.0855, 0.1246, 0.2227, 0.2878; 1)]$,

$\mathcal{C}_{21} = [(0.7885, 0.8334, 0.9345, 0.9878; 0.8), (0.7097, 0.7867, 0.9922, 1.1029; 1)]$,

$\mathcal{C}_{23} = [(0.6953, 0.7395, 0.8488, 0.9046; 0.8), (0.6113, 0.6894, 0.9102, 1.0294; 1)]$,

$\mathcal{C}_{31} = [(0.7897, 0.8347, 0.9374, 0.9980; 0.8), (0.7097, 0.7867, 0.9961, 1.1176; 1)]$,

$\mathcal{C}_{32} = [(0.5940, 0.6346, 0.7179, 0.7681; 0.8), (0.5274, 0.5956, 0.7656, 0.8676; 1)]$.

Next, the average concordance index is obtained as follows:

$\overline{\mathcal{C}} = [(0.5129, 0.5486, 0.6280, 0.6738; 0.8), (0.4503, 0.5117, 0.6732, 0.7675; 1)]$.

The discordance index is determined by means of the normalized Euclidean distances (d_E) between the weighted evaluation values. For the pair of (A_ρ, A_β), the normalized Euclidean distance $d_E(W_j \otimes A_{\rho j}, W_j \otimes A_{\beta j})$ is computed as follows:

$$d_E(W_j \otimes A_{\rho j}, W_j \otimes A_{\beta j}) = \left[\frac{1}{8}\left(\left(a_{1\rho j}^{WL} - a_{1\beta j}^{WL}\right)^2 + \left(a_{2\rho j}^{WL} - a_{2\beta j}^{WL}\right)^2 + \left(a_{3\rho j}^{WL} - a_{3\beta j}^{WL}\right)^2 + \left(a_{4\rho j}^{WL} - a_{4\beta j}^{WL}\right)^2 + \left(a_{1\rho j}^{WU} - a_{1\beta j}^{WU}\right)^2 + \left(a_{2\rho j}^{WU} - a_{2\beta j}^{WU}\right)^2 + \left(a_{3\rho j}^{WU} - a_{3\beta j}^{WU}\right)^2 + \left(a_{4\rho j}^{WU} - a_{4\beta j}^{WU}\right)^2\right)\right]^{\frac{1}{2}}. \quad (18)$$

The discordance index $\mathcal{N}_{\rho\beta}$ for each pair of (A_ρ, A_β) is defined as follows:

$$\mathcal{N}_{\rho\beta} = \max_{j \in \mathbb{N}_{\rho\beta}} d_E(W_j \otimes A_{\rho j}, W_j \otimes A_{\beta j}) / \max_{j=1}^n d_E(W_j \otimes A_{\rho j}, W_j \otimes A_{\beta j}). \quad (19)$$

The discordance index $\mathcal{N}_{\rho\beta}$ is the degree to which the weighted evaluations of A_ρ are worse than the weighted evaluations of A_β. The threshold value for $\mathcal{N}_{\rho\beta}$ is designated as the average discordance index $\overline{\mathcal{N}}$ as follows:

$$\overline{\mathcal{N}} = \sum_{\rho=1,\rho\neq\beta}^{m}\sum_{\beta=1,\beta\neq\rho}^{m} \mathcal{N}_{\rho\beta} \Big/ m(m-1). \quad (20)$$

In the medical decision problem, the discordance indices for all pairs are obtained as follows: $\mathcal{N}_{12} = 1.0000, \mathcal{N}_{13} = 1.0000, \mathcal{N}_{21} = 0.3771$, $\mathcal{N}_{23} = 1.0000$, $\mathcal{N}_{31} = 0.5998$, and $\mathcal{N}_{32} = 0.9266$. Accordingly, the average discordance index $\overline{N}$ is 0.8172.

We compute the signed distances $d(\overline{\mathcal{C}}, \tilde{0}_1)$ and $d(\mathcal{C}_{\rho\beta}, \tilde{0}_1)$ for each pair of (A_ρ, A_β). Based on the comparison of $d(\overline{\mathcal{C}}, \tilde{0}_1)$ and $d(\mathcal{C}_{\rho\beta}, \tilde{0}_1)$, the concordance dominance matrix G^1 can be constructed, and its elements are defined as follows:

$$g^1_{\rho\beta} = \begin{cases} 1 & \text{if } d(\mathcal{C}_{\rho\beta}, \tilde{0}_1) \geqslant d(\overline{\mathcal{C}}, \tilde{0}_1), \\ 0 & \text{if } d(\mathcal{C}_{\rho\beta}, \tilde{0}_1) < d(\overline{\mathcal{C}}, \tilde{0}_1). \end{cases} \quad (21)$$

Each element of one in the matrix G^1 represents a dominance of one alternative with respect to another. According to the results of the concordance indices, we have the following signed distances: $d(\mathcal{C}_{12}, \tilde{0}_1) = 0.2430$, $d(\mathcal{C}_{13}, \tilde{0}_1) = 0.3524$, $d(\mathcal{C}_{21}, \tilde{0}_1) = 1.7839$, $d(\mathcal{C}_{23}, \tilde{0}_1) = 1.6064$, $d(\mathcal{C}_{31}, \tilde{0}_1) = 1.7910$, and $d(\mathcal{C}_{32}, \tilde{0}_1) = 1.3670$. In addition, the threshold value $d(\overline{\mathcal{C}}, \tilde{0}_1) = 1.1906$. Next, the concordance dominance matrix G^1 is

$$G^1 = \begin{bmatrix} - & 0 & 0 \\ 1 & - & 1 \\ 1 & 1 & - \end{bmatrix}.$$

The discordance dominance matrix G^2 can be established by comparing the discordance index $\mathcal{N}_{\rho\beta}$ with the threshold value $\overline{\mathcal{N}}$ for each pair of (A_ρ, A_β). The elements in the matrix G^2 are defined as follows:

$$g^2_{\rho\beta} = \begin{cases} 1 & \text{if } N_{\rho\beta} \leqslant \overline{\mathcal{N}}, \\ 0 & \text{if } N_{\rho\beta} > \overline{\mathcal{N}}. \end{cases} \quad (22)$$

The unit elements in G^2 represent the dominance relationships between any two alternatives. In the medical decision problem, the discordance dominance matrix G^2 is

$$G^2 = \begin{bmatrix} - & 0 & 0 \\ 1 & - & 0 \\ 1 & 0 & - \end{bmatrix}.$$

We conduct the intersection operation of G^1 and G^2 to determine the aggregate dominance matrix $\overline{G}$. The elements in the matrix $\overline{G}$ are defined as follows:

$$\bar{g}_{\rho\beta} = g^1_{\rho\beta} \cdot g^2_{\rho\beta}. \quad (23)$$

If $\bar{g}_{\rho\beta} = 1$, then A_ρ is preferred to A_β for both the concordance and discordance criteria. In the medical decision-making problem, the aggregate dominance matrix $\overline{G}$ is obtained by combining the G^1 and G^2 matrices:

$$\overline{G} = \begin{bmatrix} - & 0 & 0 \\ 1 & - & 0 \\ 1 & 0 & - \end{bmatrix}.$$

The $\overline{G}$ matrix renders the following outranking relationships: $A_2 \succ A_1$ and $A_3 \succ A_1$. Because A_1 is dominated by A_2 and A_3, A_1 can be eliminated. However, we cannot discern the preference relation between A_2 and A_3.

The proposed extended QUALIFLEX method yielded the distinct ranking results of the alternatives: $A_2 \succ A_3 \succ A_1$. Nevertheless, the priority orders of A_2 and A_3 cannot be differentiated via the presented ELECTRE method. In addition, when the ELECTRE approach is employed within the decision environment of IT2TrFNs, the computation process is more complex and cumbersome than the presented algorithm of our proposed method. When we applied the proposed extended QUALIFLEX method to the medical decision-making problem concerning acute inflammatory demyelinating disease, the ranking result of the treatment options is reasonable and credible because the priority order of the three treatments can be clearly determined; this process makes the decision results certain and facilitates medical decision assistance and judgment. Thus, the potential of the proposed extended QUALIFLEX method to practical applications was validated through the comparative analysis.

5. Conclusions

This paper develops a new outranking method, the extended QUALIFLEX method, for handling multiple criteria decision-making problems within an IT2TrFN framework. A type-2 fuzzy approach for the expression of fuzzy linguistic variables better reflects the uncertainty of human thinking. This study applies IT2TrFNs as the alternative ratings with respect to the criteria and criterion importance weights used by the decision-maker. Compared with type-1 trapezoidal fuzzy numbers, IT2TrFNs better represent the deep-seated uncertainty manifested by the decision-maker. As the evaluative criteria become additionally complex and abstract, the IT2TrFNs become more suitable as objective and quantitative tools.

This paper makes three important contributions to the existing literature on the topic of decision-making methodology. First, the traditional QUALIFLEX method has been extended to the IT2TrFN environment to organize and model the uncertainties better within multiple criteria decision analysis. Second, instead of ordinary concordance and discordance measurements, a signed distance-based approach has been developed to identify and redefine the concordance/discordance index. Third, an algorithmic procedure for the proposed method was developed for solving decision-making problems. Finally, the real-world efficacy of the methodology was illustrated by applying the extended QUALIFLEX method to a medical decision-making problem at the Chang Gung Memorial Hospital in Taiwan.

Acknowledgments

The authors are very grateful to the respected editor and the anonymous referees for their insightful and constructive comments, which helped to improve the overall quality of the paper. The authors are grateful to the grant funding support of Taiwan National Science Council (NSC 100-2632-H-182-001-MY2) during which the study was completed. The usual disclaimer applies.

Appendix A

Definition A.1. Let X be an ordinary finite nonempty set. Let Int([0, 1]) stand for the set of all closed subintervals of [0, 1]. The mapping A: $X \to$ Int([0, 1]) is called an IT2FS on X. All IT2FSs on X are denoted by IT2FS(X).

Definition A.2. If $A \in$ IT2FS(X), let $A(x) = [A^L(x), A^U(x)]$, where $x \in X$ and $0 \leqslant A^L(x) \leqslant A^U(x) \leqslant 1$. Therefore, the two ordinary fuzzy sets A^L: $X \to [0,1]$ and A^U: $X \to [0,1]$ are called the lower and upper fuzzy sets, respectively, regarding A. If $A(x)$ is convex and is defined in a closed and bounded interval, then A is called "an interval type-2 fuzzy number (IT2FN) on X". All IT2FNs on X are denoted by IT2FN(X).

Definition A.3. Let A^L and A^U be two generalized trapezoidal fuzzy numbers, where the height of a generalized fuzzy number is between zero and one (Chen and Chen, 2009). Let h^L_A and h^U_A denote the heights of A^L and A^U, respectively. Let $a^L_1, a^L_2, a^L_3, a^L_4, a^U_1, a^U_2, a^U_3$, and

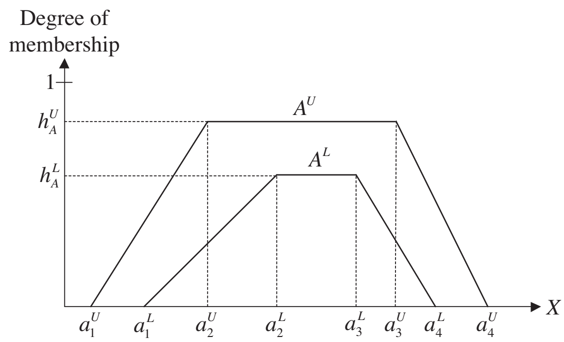

Fig. A.1. An IT2TrFN A.

a_4^U be real values. An IT2TrFN A (see Fig. A.1) defined on the universe of discourse X is represented by the following:

$$A = [A^L, A^U] = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; h_A^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; h_A^U\right)\right], \tag{A.1}$$

where $a_1^L \leqslant a_2^L \leqslant a_3^L \leqslant a_4^L, a_1^U \leqslant a_2^U \leqslant a_3^U \leqslant a_4^U$, $0 \leqslant h_A^L \leqslant h_A^U \leqslant 1, a_1^U \leqslant a_1^L$, and $a_4^L \leqslant a_4^U$. The lower trapezoidal fuzzy number $A^L = (a_1^L, a_2^L, a_3^L, a_4^L; h_A^L)$ and the upper trapezoidal fuzzy number $A^U = \left(a_1^U, a_2^U, a_3^U, a_4^U; h_A^U\right)$, where $A^L \subset A^U$.

Definition A.4. The arithmetic operations between the two non-negative IT2TrFNs $A = [A^L, A^U] = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; h_A^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; h_A^U\right)\right]$ and $B = [B^L, B^U] = \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; h_B^L\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; h_B^U\right)\right]$ are defined as follows:

$$A \oplus B = \left[\left(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; \min\left(h_A^L, h_B^L\right)\right), \left(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U; \min\left(h_A^U, h_B^U\right)\right)\right]; \tag{A.2}$$

$$A \Theta B = \left[\left(a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L; \min\left(h_A^L, h_B^L\right)\right), \left(a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U; \min\left(h_A^U, h_B^U\right)\right)\right]; \tag{A.3}$$

$$A \otimes B = \left[\left(a_1^L \times b_1^L, a_2^L \times b_2^L, a_3^L \times b_3^L, a_4^L \times b_4^L; \min\left(h_A^L, h_B^L\right)\right), \left(a_1^U \times b_1^U, a_2^U \times b_2^U, a_3^U \times b_3^U, a_4^U \times b_4^U; \min\left(h_A^U, h_B^U\right)\right)\right]; \tag{A.4}$$

$$A \emptyset B = \left[\left(a_1^L/b_4^L, a_2^L/b_3^L, a_3^L/b_2^L, a_4^L/b_1^L; \min\left(h_A^L, h_B^L\right)\right), \left(a_1^U/b_4^U, a_2^U/b_3^U, a_3^U/b_2^U, a_4^U/b_1^U; \min\left(h_A^U, h_B^U\right)\right)\right], \quad b_1^L, b_2^L, b_3^L, b_4^L, b_1^U, b_2^U, b_3^U, b_4^U \neq 0; \tag{A.5}$$

$$q \cdot A = A \cdot q = \begin{cases} \left[\left(q \times a_1^L, q \times a_2^L, q \times a_3^L, q \times a_4^L; h_A^L\right), \left(q \times a_1^U, q \times a_2^U, q \times a_3^U, q \times a_4^U; h_A^U\right)\right] & \text{if } q \geqslant 0, \\ \left[\left(q \times a_4^L, q \times a_3^L, q \times a_2^L, q \times a_1^L; h_A^L\right), \left(q \times a_4^U, q \times a_3^U, q \times a_2^U, q \times a_1^U; h_A^U\right)\right] & \text{if } q \leqslant 0; \end{cases} \tag{A.6}$$

$$\frac{A}{q} = \begin{cases} \left[\left(a_1^L/q, a_2^L/q, a_3^L/q, a_4^L/q; h_A^L\right), \left(a_1^U/q, a_2^U/q, a_3^U/q, a_4^U/q; h_A^U\right)\right] & \text{if } q > 0, \\ \left[\left(a_4^L/q, a_3^L/q, a_2^L/q, a_1^L/q; h_A^L\right), \left(a_4^U/q, a_3^U/q, a_2^U/q, a_1^U/q; h_A^U\right)\right] & \text{if } q < 0. \end{cases} \tag{A.7}$$

Appendix B

See Table B.1.

Table B.1
The mathematical notation.

Symbols	The mathematical meanings
A	The set of decision alternatives
A_i	The element in A (i = 1, 2, ..., m)
X	The set of criteria
x_j	The element in X (j = 1, 2, ..., n)
A_{ij}	The evaluation of alternative A_i on criterion x_j
D	The decision matrix
W_j	The importance weight of criterion x_j
$\tilde{0}_1$	The level 1 fuzzy number mapping onto the y-axis
$d(A_{ij}, \tilde{0}_1)$	The signed distance from A_{ij} to $\tilde{0}_1$
P_l	The lth permutation of the alternatives
$I_j^l(A_\rho, A_\beta)$	The concordance/discordance index for the pair of (A_ρ, A_β)
I_j^l	The concordance/discordance index for P_l w.r.t. x_j
$I^l(A_\rho, A_\beta)$	The weighted concordance/discordance index
I^l	The comprehensive concordance/discordance index for P_l
$\mathbb{C}_{\rho\beta}$	The concordance set of the pair of (A_ρ, A_β)
$\mathbb{N}_{\rho\beta}$	The discordance set of the pair of (A_ρ, A_β)
$\mathcal{C}_{\rho\beta}$	The concordance index of the pair of (A_ρ, A_β)
$\mathcal{N}_{\rho\beta}$	The discordance index of the pair of (A_ρ, A_β)
G^1	The concordance dominance matrix
G^2	The discordance dominance matrix
$\overline{G}$	The aggregate dominance matrix

References

Abbasbandy, S., Asady, B., 2006. Ranking of fuzzy numbers by sign distance. Information Sciences 176, 2405–2416.
Benayoun, R., Roy, B., Sussman, N., 1966. Manual de Reference du Program ELECTRE. Note de Synthese et Formation, Direction Scientifique SEMA, No. 25, Paris, France.
Brailsford, S.C., Harper, P.R., Sykes, J., 2012. Incorporating human behaviour in simulation models of screening for breast cancer. European Journal of Operational Research 219, 491–507.
Brans, J.P., Mareschal, B., Vincke, Ph., 1984. PROMETHEE: a new family of outranking methods in MCDM. In: Operational Research, IFORS'84. North Holland, pp. 477–490.
Büyüközkan, G., Çifçi, G., 2012. A combined fuzzy AHP and fuzzy TOPSIS based strategic analysis of electronic service quality in healthcare industry. Expert Systems with Applications 39, 2341–2354.
Chen, C.-T., 2000. Extensions of the TOPSIS for group decision-making under fuzzy environment. Fuzzy Sets and Systems 114, 1–9.
Chen, T.-Y., 2011a. Signed distanced-based TOPSIS method for multiple criteria decision analysis based on generalized interval-valued fuzzy numbers. International Journal of Information Technology and Decision Making 10, 1131–1159.
Chen, T.-Y., 2011b. An integrated approach for assessing criterion importance with interval type-2 fuzzy sets and signed distances. Journal of the Chinese Institute of Industrial Engineers 28, 553–572.
Chen, T.-Y., 2012. Multiple criteria group decision-making with generalized interval-valued fuzzy numbers based on signed distances and incomplete weights. Applied Mathematical Modelling 36, 3029–3052.
Chen, S.-J., Chen, S.-M., 2008. Fuzzy risk analysis based on measures of similarity between interval-valued fuzzy numbers. Computers and Mathematics with Applications 55, 1670–1685.
Chen, S.-M., Chen, J.-H., 2009. Fuzzy risk analysis based on similarity measures between interval-valued fuzzy numbers and interval-valued fuzzy number arithmetic operators. Expert Systems with Applications 36, 6309–6317.
Chen, S.-M., Lee, L.-W., 2010a. Fuzzy multiple attributes group decision-making based on the ranking values and the arithmetic operations of interval type-2 fuzzy sets. Expert Systems with Applications 37, 824–833.
Chen, S.-M., Lee, L.W., 2010b. Fuzzy multiple criteria hierarchical group decision-making based on interval type-2 fuzzy sets. IEEE Transactions on Systems, Man, and Cybernetics Part A: Systems and Humans 40, 1120–1128.
Chen, L.-H., Ouyang, L.-Y., 2006. Fuzzy inventory model for deteriorating items with permissible delay in payment. Applied Mathematics and Computation 182, 711–726.

Chen, T.-Y., Wang, J.-C., 2009. Interval-valued fuzzy permutation method and experimental analysis on cardinal and ordinal evaluations. Journal of Computer and System Sciences 75, 371–387.
Chen, S.-M., Yang, M.-W., Lee, L.-W., Yang, S.-W., 2012. Fuzzy multiple attributes group decision-making based on ranking interval type-2 fuzzy sets. Expert Systems with Applications 39, 5295–5308.
Chiang, J., 2001. Fuzzy linear programming based on statistical confidence interval and interval-valued fuzzy set. European Journal of Operational Research 129, 65–86.
Creemers, S., Beliën, J., Lambrecht, M., 2012. The optimal allocation of server time slots over different classes of patients. European Journal of Operational Research 219, 508–521.
Dekker, S., 2012. Complexity, signal detection, and the application of ergonomics: reflections on a healthcare case study. Applied Ergonomics 43, 468–472.
Dursun, M., Karsak, E.E., Karadayi, M.A., 2011. A fuzzy multi-criteria group decision making framework for evaluating health-care waste disposal alternatives. Expert Systems with Applications 38, 11453–11462.
Esposito, M., De Falco, I., De Pietro, G., 2011. An evolutionary-fuzzy DSS for assessing health status in multiple sclerosis disease. International Journal of Medical Informatics 80, e245–e254.
Fernandez, E., Navarro, J., 2011. A new approach to multi-criteria sorting based on fuzzy outranking relations: the THESEUS method. European Journal of Operational Research 213, 405–413.
Figueira, J., Greco, S., Ehrgott, M., 2005. Multiple Criteria Decision Analysis: State of the Art Surveys. Springer Science + Business Media, New York.
Gilan, S.S., Sebt, M.H., Shahhosseini, V., 2012. Computing with words for hierarchical competency based selection of personnel in construction companies. Applied Soft Computing 12, 860–871.
Greenfield, S., Chiclana, F., Coupland, S., John, R., 2009. The collapsing method of defuzzification for discretised interval type-2 fuzzy sets. Information Sciences 179, 2055–2069.
Griffith, D.A., Paelinck, J.H.P., 2011. Qualireg, a qualitative regression method. Advances in Geographic Information Science 1, 227–233.
Grosan, C., Abraham, A., Tigan, S., 2008. Multicriteria programming in medical diagnosis and treatments. Applied Soft Computing 8, 1407–1417.
Hwang, C.L., Yoon, K., 1981. Multiple Attribute Decision Making: Methods and Applications. Springer-Verlag, Berlin/Heidelberg/New York.
Ippoliti, R., Falavigna, G., 2012. Efficiency of the medical care industry: evidence from the Italian regional system. European Journal of Operational Research 217, 643–652.
Koulouriotis, D.E., Mantas, G., 2012. Health products sales forecasting using computational intelligence and adaptive neuro fuzzy inference systems. Operational Research 12, 29–43.
Lahdelma, R., Miettinen, K., Salminen, P., 2003. Ordinal criteria in stochastic multicriteria acceptability analysis (SMAA). European Journal of Operational Research 147, 117–127.
Lee, Y.-Y., Lin, J.L., 2010. Do patient autonomy preferences matter? Linking patient-centered care to patient–physician relationships and health outcomes. Social Science and Medicine 71, 1811–1818.
Liberatore, M.J., Nydick, R.L., 2008. The analytic hierarchy process in medical and health care decision making: a literature review. European Journal of Operational Research 189, 194–207.
Lutz, B.J., Bowers, B.J., 2000. Patient-centered care: understanding its interpretation and implementation in health care. Scholarly Inquiry for Nursing Practice 14, 165–182.
Mehrotra, S., Kim, K., 2011. Outcome based state budget allocation for diabetes prevention programs using multi-criteria optimization with robust weights. Health Care Management Science 14, 324–337.
Mendel, J.M., John, R.I., Liu, F., 2006. Interval type-2 fuzzy logic systems made simple. IEEE Transactions on Fuzzy Systems 14, 808–821.
Merad, M., Dechy, N., Serir, L., Grabisch, M., Marcel, F., 2013. Using a multi-criteria decision aid methodology to implement sustainable development principles within an organization. European Journal of Operational Research 224, 603–613.
Moreno, E., Girón, F.J., Vázquez-Polo, F.J., Negrín, M.A., 2010. Optimal healthcare decisions: comparing medical treatments on a cost-effectiveness basis. European Journal of Operational Research 204, 180–187.
Moreno, E., Girón, F.J., Vázquez-Polo, F.J., Negrín, M.A., 2012. Optimal healthcare decisions: the importance of the covariates in cost-effectiveness analysis. European Journal of Operational Research 218, 512–522.
Paelinck, J.H.P., 1976. Qualitative multiple criteria analysis, environmental protection and multiregional development. Papers of the Regional Science Association 36, 59–74.
Rebai, A., Aouni, B., Martel, J.-M., 2006. A multi-attribute method for choosing among potential alternatives with ordinal evaluation. European Journal of Operational Research 174, 360–373.
Sarabando, P., Dias, L.C., 2010. Simple procedures of choice in multicriteria problems without precise information about the alternatives' values. Computers and Operations Research 37, 2239–2247.
Šušteršič, O., Rajkovič, U., Dinevski, D., Jereb, E., Rajkovič, V., 2009. Evaluating patients' health using a hierarchical multi-attribute decision model. Journal of International Medical Research 37, 1646–1654.
Tamanini, I., De Castro, A.K., Pinheiro, P.R., Pinheiro, M.C.D., 2009. Applied neuroimaging to the diagnosis of alzheimer's disease: a multicriteria model. Communications in Computer and Information Science 49, 532–541.
Tartarisco, G., Baldus, G., Corda, D., Raso, R., Arnao, A., Ferro, M., Gaggioli, A., Pioggia, G., 2012. Personal health system architecture for stress monitoring and support to clinical decisions. Computer Communications 35, 1296–1305.
Tsai, H.-Y., Chang, C.-W., Lin, H.-L., 2010. Fuzzy hierarchy sensitive with Delphi method to evaluate hospital organization performance. Expert Systems with Applications 37, 5533–5541.
Vahdani, B., Hadipour, H., 2011. Extension of the ELECTRE method based on interval-valued fuzzy sets. Soft Computing 15, 569–579.
Vahdani, B., Jabbari, A.H.K., Roshanaei, V., Zandieh, M., 2010. Extension of the ELECTRE method for decision-making problems with interval weights and data. International Journal of Advanced Manufacturing Technology 50, 793–800.
Wang, W., Liu, X., Qin, Y., 2012. Multi-attribute group decision making models under interval type-2 fuzzy environment. Knowledge-Based Systems 30, 121–128.
Wei, S.-H., Chen, S.-M., 2009. Fuzzy risk analysis based on interval-valued fuzzy numbers. Expert Systems with Applications 36, 2285–2299.
Wu, D., Mendel, J.M., 2007. Uncertainty measures for interval type-2 fuzzy sets. Information Sciences 177, 5378–5393.
Yao, J.S., Wu, K., 2000. Ranking fuzzy numbers based on decomposition principle and signed distance. Fuzzy Sets and Systems 116, 275–288.
Yucel, G., Cebi, S., Hoege, B., Ozok, A.F., 2012. A fuzzy risk assessment model for hospital information system implementation. Expert Systems with Applications 39, 1211–1218.
Zadeh, L.A., 1975. The concept of a linguistic variable and its application to approximate reasoning-I. Information Sciences 8, 199–249.