This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

Generation Capacity Expansion in Liberalized Electricity Markets: A Stochastic MPEC Approach

Sonja Wogrin, Efraim Centeno, and Julián Barquín, *Member, IEEE*

***Abstract*—This paper proposes a bilevel model to assist a generation company in making its long-term generation capacity investment decisions considering uncertainty regarding the investments of the other generation companies. The bilevel formulation allows for the uncoupling of investment and generation decisions, as investment decisions of the single investing generation company are taken in the upper level with the objective to maximize expected profits and generation decisions by all companies are considered in the lower level. The lower level represents the oligopolistic market equilibrium via a conjectured-price response formulation, which can capture various degrees of strategic market behavior like perfect competition, the Cournot oligopoly, and intermediate cases.**

The bilevel model is formulated as an MPEC, replacing the lower level by its KKT conditions and transformed into a MILP. Results from a study case are presented and discussed.

***Index Terms*—Bilevel programming, generation expansion planning, mathematical program with equilibrium constraints (MPEC).**

I. Introduction

DUE to the liberalization of electricity markets, the task of taking generation capacity investment decisions has become an even more complex problem than it already has been under a centralized framework. Generation companies are now exposed to a higher level of risk, having to deal with the strategic behavior of competitors in imperfect markets and coping with the uncertainty due to regulatory decisions, fuel prices, demand, competitive behavior in the electricity market, and hydro inflows, among others.

In regulated frameworks, decisions regarding generation capacity expansion were generally easier to predict and therefore relatively stable, as they were usually made by a centralized planner. The main techniques to model these kinds of decisions are multi-criteria decision methods [1] and optimization methods, which often boil down to a cost minimization problem. For further detail, the reader is referred to [2].

But in liberalized systems, the generation capacity planning becomes a more elusive problem because of additional uncertainty due to competition. We can distinguish between methods that focus on the improvement of the uncertainty treatment, i.e., scenario analysis, decision theory, risk management, and real options theory and techniques that focus on the analysis of markets and the behavior of its competitors. The corresponding methods are game theory, multi-agent based simulation, and system dynamics. For further information on the previously mentioned methods and detailed references, refer to [3].

In this paper, we will focus on game-theoretic methods in liberalized frameworks, in particular on bilevel formulations. In general, a mathematical programming problem is classified as a bilevel programming problem when one of the constraints of an optimization problem is also an optimization problem. Another class of problems closely related to bilevel programs are mathematical programs with equilibrium constraints (MPECs) which have been applied in various fields like engineering, economics, and finance. An MPEC is an optimization problem in which the essential constraints are defined by a parametric variational inequality [4] or a complementarity system [5], which typically model a certain equilibrium phenomenon. In the electricity sector, MPECs have first been used to formulate electricity markets by [6] and [7].

Game theory is particularly useful in the energy sector because it allows us to analyze the strategic behavior of agents—in our case generation companies—whose interests are opposing and whose decisions influence the outcome of other agents. The complex of problems, where each generation company faces a MPEC when deciding its capacity expansion in order to maximize profits taking into account that the other companies will do the same, can be interpreted as a game among generation companies and can hence be formulated as an equilibrium problem with equilibrium constraints (EPEC), see [8] and [9], which will be investigated in future research.

Within the game-theoretic framework, one approach for the capacity expansion problem is to extend medium-term models to longer terms, by considering that investment and production decisions are taken at the same time. This corresponds to the open-loop equilibrium conditions presented in [10], the Cournot-based model presented in [11], which is solved using a mixed complementarity problem (MCP) schema, and the model analyzed in [12], which is solved using an equivalent optimization problem. However, this approach has its drawbacks, as it may overly simplify the dynamic nature of the problem, as expansion and operation decisions are taken simultaneously.

Uncoupling expansion and operation decisions leads to the more complex bilevel modeling. An instance of this modeling approach is the closed-loop Cournot game described in [10], where qualitative properties of the model are investigated. In this case, the existence and uniqueness of this kind of equilibrium are not guaranteed in most situations (see [9] for a discussion on the topic). Moreover, the equilibrium approach must

Manuscript received December 09, 2010; revised February 25, 2011; accepted March 29, 2011. Paper no. TPWRS-00995-2010.

The authors are with the Universidad Pontificia Comillas (ETS de Ingeniería ICAI. Instituto de Investigación Tecnológica), 28015 Madrid, Spain (e-mail: Sonja.Wogrin@iit.upcomillas.es; efraim.centeno@iit.upcomillas.es; julian.barquin@iit.upcomillas.es).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TPWRS.2011.2138728

be built on the definition of single-agent optimality. Even the single-agent optimality poses a non-convex problem, as it is also subject to the spot market equilibrium, which defines a non-convex set. A Stackelberg-like bilevel game between co-generators can be found in [13], and in [11], we find a closed-loop Stackelberg-based model which is formulated as an MPEC. In [14], the authors present a linear bilevel model under uncertainty that determines the optimal investment decisions of a generation company assuming a perfectly competitive market. An instance of a stochastic static bilevel model for the generation capacity problem can be found in [15], where investment and production decisions are taken in the upper level for a single target year in the future, while the lower level represents the market clearing.

In this paper, we would like to address some of the shortcomings of existing approaches in the literature. First of all, even though existing open-loop approaches like in [10]–[12] are adequate and useful to approximate the generation capacity problem, they do not model the significant temporal separation between when capacity decisions are taken and when energy is produced with that capacity. We overcome this problem by proposing a bilevel model. Furthermore, existing bilevel approaches in the literature assume either perfectly competitive [14] or Cournot behavior [11], [13] in the spot market. We want to extend these approaches to also capture intermediate oligopolistic behavior in order to explore how capacity decisions would change if competitive behavior in the spot market changed. The model presented in this paper represents the market via a conjectured price response formulation, thereby allowing us to model a range of oligopolistic market behavior. Finally, the model yields an investment schedule over the entire time horizon, as opposed to a static investment decision for a future target year, as done in [15].

In particular, in this paper, a stochastic bilevel model is proposed which takes the point of view of a single investing agent faced with generation capacity expansion decisions, which is transformed into a stochastic MPEC. In the upper level, the investing agent maximizes its expected profit deciding its generation capacity. The lower level problem corresponds to a conjectural variations market equilibrium closely inspired by [16] but including conjectured price variations as uncertain parameters, which can be formulated as a convex optimization problem corresponding to [17]. Replacing the lower level by its Karush-Kuhn-Tucker (KKT) conditions yields an MPEC.

Furthermore uncertainty regarding the investment decisions of the competition and a corresponding strategic behavior on the market is introduced. As investment decisions crucially depend on the strategic behavior of the generation companies in the market, which we model using a conjectured-price-variation approach, we incorporate a set of scenarios of conjectured-price responses into the generation capacity expansion problem, each representing a different realization of possible market behavior. As the uncertainty corresponding to competition is being considered as the most driving factor, additional sources of uncertainty, such as fuel prices or demand growth, are not yet considered and will be topics for future research.

The proposed stochastic MPEC is then solved using two methods: the first one is to apply a nonlinear solver to the MPEC and the second one includes transforming the MPEC into a mixed integer linear problem (MILP), which is then solved using mixed integer programming.

This paper is organized as follows. In Section II, the proposed bilevel model is described, then the conjectured-price response market representation is motivated and the stochastic framework is presented. In Section III, the proposed stochastic bilevel model is formulated. Section IV provides a case study to analyze the presented model. Finally, in Section V, some relevant conclusions are drawn.

II. Problem Description

In this section, the generation capacity expansion problem as a bilevel program is briefly described, then the conjectured-price-response market representation and corresponding equilibrium conditions are presented, and finally the stochastic framework is incorporated into the problem.

A. Bilevel Problem Description

In the generation capacity expansion model, which is presented in this paper, one generation company aims at maximizing possible profits by deciding its investments in generation capacity, while the investment capacity of the other generation companies is introduced via a set of scenarios. These capacities determine the maximum possible amount of energy production in the market. It is easy to see that this problem has an innate two-stage structure: first investment decisions are taken and then energy productions, which are limited by the previously decided capacity, are determined by a market clearing. Hence the generation capacity problem can be modeled as a bilevel program, where the upper level objective function corresponds to the profit maximization of a generation company and the lower level corresponds to the market equilibrium between all participating generation companies, which can be formulated as an optimization problem as shown in [17].

The bilevel structure of this model underlines the opposing interest of the investing generation company and the market, as both interests are embodied by different objective functions. Moreover the bilevel formulation allows for the decoupling of investment and production decisions, which in reality are not taken at the same time but which are linked through the installed generation capacity, a fact that can literally be observed in the problem formulation.

B. Conjectured-Price-Response Market Representation

The market is represented via a conjectured-price-response approach like the one found in [16] and [17], which allows us to model the strategic behavior of generation companies in an oligopolistic market. Let us now summarize the most important points to build this model. First of all, demand d will be considered to be affine as $d = E - \alpha p$, where α is the demand slope and E the demand intercept. The demand slope and intercept are considered input data whereas demand and price are outputs. Demand and productions are linked by $\sum_i q_i = d$.

Let us now introduce a conjectured partial derivative, which expresses a company's belief about its influence on price as a result of a change in its output–a conjectured price response. Every company assumes that there is a certain change in price when changing their quantities, which is captured by the conjectured price response parameter θ_i, which is defined as follows:

$$\theta_i = -\frac{\partial p}{\partial q_i} \geq 0. \tag{1}$$

This parameter is assumed to be known for all generation companies. This formulation includes perfect competition ($\theta_i = 0$) and the Cournot oligopoly ($\theta_i = 1/\alpha$) as special cases. The individual profit π_i of each company can be obtained by $\pi_i = pq_i - C_i(q_i)$, where the term pq_i corresponds to the revenues and $C_i(q_i)$ corresponds to the production cost function. Then the market equilibrium can be obtained by the following set of conditions using (1):

$$\begin{aligned}\frac{\partial \pi_i}{\partial q_i} &= p + q_i\frac{\partial p}{\partial q_i} - \frac{\partial C_i(q_i)}{\partial q_i}\\ &= p - q_i\theta_i - \frac{\partial C_i(q_i)}{\partial q_i} = 0 \quad \forall i.\end{aligned} \tag{2}$$

Considering the convexity and continuity of all cost functions, the market equilibrium can be written as the following optimization problem, which was proven in [17]:

$$\begin{aligned}\min_{q_i,d} \quad & \sum_i \hat{C}_i(q_i) - \vartheta(d)\\ \text{s.t.} \quad & \sum_i q_i = d \quad : p\\ & \text{Technical Constraints}\end{aligned} \tag{3}$$

where $\hat{C}_i(q_i) = C_i(q_i) + q_i^2\theta_i/2$ is called the extended cost function (cost plus quadratic terms) and $\vartheta(d)$ is the demand utility defined as $\vartheta(d) = \int_0^d p(d)\mathbf{d}d = 1/\alpha(Ed - d^2/2)$. The Technical Constraints depend on the particular application and can lead to different effects on the equilibrium, which are discussed in [18].

C. Stochastic Framework

The choice of capacity investments of the investing generation company mainly depends on two factors: the investment of the competing generation companies and the strategic behavior in the market, which determines prices and productions. As these two factors are by no means independent of each other, uncertainty has been introduced through a set of scenarios s of investment capacities of competitors coupled with corresponding conjectured price responses.

The estimation of the conjectured price response is typically based on historical data and there exist implicit methods [19] (adjusting past market prices) and explicit methods [20]. For a summary of conjectural variations estimation methods, the reader is referred to [21]. The computation of the value of conjectured price responses will not be analyzed in the present article. Reasonable values for these parameters, representing different degrees of oligopoly, will be considered as known.

Then these scenarios are introduced in the formulation of the market equilibrium. Hence all lower level variables like productions q_i, demand d, and price p will be stochastic variables and therefore depend on the chosen scenario. The upper level variables, which correspond to the installed generation capacity of the investing agent, however will not be stochastic variables, because a generation company can only make one investment decision as it is impossible to know which scenario is going to occur in reality. In order to incorporate the uncertainty in the upper level of our bilevel model, the criterion of maximizing the expected value to the upper level objective function is applied, i.e., the investing generation company will now be maximizing the expected profits considering all given scenarios. This implies that the investing firm is assumed to be risk neutral.

III. Model Formulation

First of all, we define all the indices that will be used throughout the formulation of the model. The index y corresponds to the set of years of the time horizon for which we make our investment decisions; l corresponds to the load level with duration t of each year in the time scope; i is the index of all generation companies; by i^*, we denote the investing generation company and by $-i^*$ we denote all generation companies excluding the investing company; j corresponds to the different technologies of generation capacity; s is the set of all scenarios; and finally let $\tilde{s}$ denote one of these scenarios.

A. Formulation of the Lower Level

For one scenario $\tilde{s}$, the lower level is presented in (4)–(7) and represents the conjectured-price-response market equilibrium of (2), formulated as an optimization problem as detailed in (3). It can be shown that the arising optimization problem is convex. In this section, the lower level model will be presented for one scenario; however, the reader should keep in mind that in the resulting bilevel model, the lower level model has to be solved for every scenario s.

Let us now describe the lower level model under scenario $\tilde{s}$ in detail. The decision variables of the lower level are given by: the production $q_{ijyl\tilde{s}}$[GW] of each agent i, of each technology j, in each load level l of each year y in scenario $\tilde{s}$ and the demand $d_{yl\tilde{s}}$[GW] in each load level l and year y depending on the current scenario $\tilde{s}$:

$$\begin{aligned}\min_{q,d} \quad & \sum_{ijyl} \delta_j q_{ijyl\tilde{s}} t_{yl} + \frac{1}{2}\sum_{iyl} \theta_{iyl\tilde{s}} t_{yl}\left(\sum_j q_{ijyl\tilde{s}}\right)^2\\ & - \sum_{yl} \frac{t_{yl}}{\alpha_{yl}}\left(E_{yl}d_{yl\tilde{s}} - \frac{d^2_{yl\tilde{s}}}{2}\right) & (4)\\ \text{s.t.} \quad & 0 \leq q_{i^*jyl\tilde{s}} \leq x_{i^*jy} \quad : \mu_{i^*jyl\tilde{s}}, \lambda_{i^*jyl\tilde{s}} & (5)\\ & 0 \leq q_{-i^*jyl\tilde{s}} \leq x_{-i^*jy\tilde{s}} \quad : \mu_{-i^*jyl\tilde{s}}, \lambda_{-i^*jyl\tilde{s}} & (6)\\ & \sum_{ij} t_{yl} q_{ijyl\tilde{s}} = t_{yl} d_{yl\tilde{s}} \quad : p_{yl\tilde{s}}. & (7)\end{aligned}$$

The lower level objective function, which corresponds to the one in (3), is stated in (4) and yields the market equilibrium by minimizing the difference between the extended costs minus the demand utility—both of which have already been defined in Section II-B, i.e., solving the market clearing. The extended costs are computed using the constants δ_j[€/MWh] which correspond to unitary production costs of each technology j, the duration of each load level t_{yl} [Kh], and the conjectured price

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

responses $\theta_{iyl\tilde{s}}[(€/\mathrm{MW})/\mathrm{GW}]$ of each company i in each load level l of each year y in scenario $\tilde{s}$.

The constraints that are linking the lower and the upper level can be found in (5) and (6), which correspond to the lower and upper bounds of production $q_{ijyl\tilde{s}}$. For the investing agent, the production is limited from above by the upper level variable x_{i^*jy}, see (5), which is the generation capacity in technology j and year y. For the non-investing companies, however, the productions are limited from above, see (6), by the constants $x_{-i^*jy\tilde{s}}$, which depend on the scenario $\tilde{s}$.

Equation (7) represents the power-demand balance equation that we have also seen in problem (3). Note that the dual variable of this equation $p_{yl\tilde{s}}$ corresponds to the system's marginal price that clears the market. Similarly, $\mu_{ijyl\tilde{s}}, \lambda_{ijyl\tilde{s}}$ are dual variables corresponding to the constraints given in (5)–(6).

B. Formulation of the Upper Level

The formulation of the upper level can be found in (8)–(9), which corresponds to the "leave it to the market" approach. The presented methodology could be adapted in order to incorporate a different capacity mechanism, e.g., capacity markets or capacity payments, by adding a new term corresponding to the capacity revenues in the objective function. However, under the "leave it to the market" approach, in the upper level objective function, which can be found in (8), we maximize the expected value of profit, i.e., net present value of company i^* which is the only company deciding its investment in generation capacity x_{i^*jy} of each technology j in each year y. Note that the capacities of the rest of the companies x_{-i^*jys} are not decision variables of this problem and are incorporated via the scenarios s. In (8), we sum over the net present value, discounted with discount rate f, obtained in each scenario s multiplied by the probability of this scenario w_s.

In general the profit π_{i^*} of company i^* corresponds to the market revenues minus the arising costs. The market revenues are given by the product between market price p_{yls} and productions q_{i^*jyls}. The costs consist of production costs and investment costs. The production costs correspond to the term $\delta_j q_{i^*jyls}$ and the investment costs are given by the term $\beta_{i^*jy}x_{i^*jy}$, where $\beta_{i^*jy}[M€/\mathrm{GW/yr}]$ is the unitary annual investment cost. An additional monotonicity of generation capacity is fulfilled in (9):

$$\max_{x_{i^*jy}} \quad \sum_s w_s \left\{ \sum_y \frac{1}{(1+f)^y} \left[\sum_{jl} t_{yl} p_{yls} q_{i^*jyls} - \sum_{jl} t_{yl}\delta_j q_{i^*jyls} - \sum_j \beta_{i^*jy} x_{i^*jy} \right] \right\} \tag{8}$$

$$\text{s.t.} \quad 0 \le x_{i^*j(y-1)} \le x_{i^*jy}. \tag{9}$$

C. Reformulation as MPEC

The bilevel formulation of the generation capacity expansion problem is given by the upper level subject to the lower level for each scenario s. As the lower level optimization problem given by (4)–(7) is convex and continuous, we can replace it by its KKT conditions and rewrite the proposed bilevel problem as an MPEC given in (10)–(19), where (10) and (11) is the part corresponding to the former upper level, (12)–(13) are the derivatives of the Lagrangian of the lower level, (14)–(16) are the complementarity conditions, and (17)–(19) are the constraints of the lower level:

$$\max_{x_{i^*},q,d,p} \quad \sum_s w_s \left\{ \sum_y \frac{1}{(1+f)^y} \left[\sum_{jl} t_{yl} p_{yls} q_{i^*jyls} - \sum_{jl} t_{yl}\delta_j q_{i^*jyls} - \sum_j \beta_{i^*jy} x_{i^*jy} \right] \right\} \tag{10}$$

$$\text{s.t.} \quad 0 \le x_{i^*j(y-1)} \le x_{i^*jy} \quad \forall jy \tag{11}$$

$$\delta_j t_{yl} + \sum_j \theta_{iyls} q_{ijyls} t_{yl} + \lambda_{ijyls} - \mu_{ijyls} - t_{yl} p_{yls} = 0 \quad \forall ijyls \tag{12}$$

$$-t_{yl}(E_{yl} - d_{yls}) + \alpha_{yl} t_{yl} p_{yls} = 0 \quad \forall yls \tag{13}$$

$$\mu_{ijyls} q_{ijyls} = 0 \quad \forall ijyls \tag{14}$$

$$\lambda_{i^*jyls}(x_{i^*jy} - q_{i^*jyls}) = 0 \quad \forall jyls \tag{15}$$

$$\lambda_{-i^*jyls}(x_{-i^*jys} - q_{-i^*jyls}) = 0 \quad \forall -i^*jyls \tag{16}$$

$$0 \le q_{i^*jyls} \le x_{i^*jy} \quad \forall jyls \tag{17}$$

$$0 \le q_{-i^*jyls} \le x_{-i^*jys} \quad \forall -i^*jyls \tag{18}$$

$$\sum_{ij} t_{yl} q_{ijyls} = t_{yl} d_{yls} \quad \forall yls. \tag{19}$$

D. Resolution Methods

1) Nonlinear Programming: The presented MPEC is a nonlinear non-convex problem, due to the complementarities and the bilinear product of prices and quantities in the objective function. Therefore employing a nonlinear solver does not guarantee to find a global optimum. However, smart initialization may assist the nonlinear solver to find a satisfactory solution. In order to obtain a meaningful initial solution to the MPEC, we first solve a simultaneous optimization problem, i.e., investment and production decisions are taken simultaneously, such as the one given in [11, Appendix B], which is formulated as an MCP.

2) Mixed Integer Programming: As a second resolution approach, we linearize the presented MPEC to obtain a MILP. As stated above, the nonlinearities of the MPEC are due to the complementarities and the product of prices and quantities in the objective function. We take care of the complementarities (14)–(16) by replacing them by their linear equivalent, see [22]:

$$C^\mu b^\mu_{ijyls} \ge \mu_{ijyls}, \tag{20}$$

$$C^\mu(1 - b^\mu_{ijyls}) \ge q_{ijyls}, \tag{21}$$

$$C^\lambda b^\lambda_{i^*jyls} \ge \lambda_{i^*jyls}, \tag{22}$$

$$C^\lambda(1 - b^\lambda_{i^*jyls}) \ge (x_{i^*jy} - q_{i^*jyls}), \tag{23}$$

$$C^\lambda b^\lambda_{-i^*jyls} \ge \lambda_{-i^*jyls}, \tag{24}$$

$$C^\lambda(1 - b^\lambda_{-i^*jyls}) \ge (x_{-i^*jys} - q_{-i^*jyls}) \tag{25}$$

for C^μ, C^λ suitably large constants and $b^\mu_{ijyls}, b^\lambda_{ijyls}$ binary variables.

As for the bilinear terms $p_{yls}q_{i^*jyls}$ in the objective function, we apply a binary expansion to the variable price p_{yls}, see [23]:

TABLE I
INITIAL INSTALLED GENERATION CAPACITY OF FIRMS 2 AND 3 FOR ALL SCENARIOS [MW]

Technology	s_1	s_2	s_3
Nuclear	8000	6210	4520
CCGT	2670	2070	1510

TABLE II
PROBABILITY [P.U.] AND CONJECTURED PRICE RESPONSE $\theta[(€/\mathrm{MW})/\mathrm{GW}]$ OF SCENARIOS

	s_1	s_2	s_3
w_s	0.2	0.5	0.3
θ	0	2.175	4.35

$$p_{yls} = \underline{p}_{yls} + \Delta_{p_{yls}} \sum_k 2^k b^p_{kyls} \tag{26}$$

where $\underline{p}_{yls}$ is the lower bound, $\Delta_{p_{yls}}$ is the step size of price, k the set of discretization intervals, and b^p_{kyls} are binary variables. Then the bilinear terms $p_{yls}q_{i^*jyls}$ of the objective can be replaced by $\underline{p}_{yls}q_{i^*jyls} + \Delta_{p_{yls}} \sum_k 2^k z_{ki^*jyls}$, where z_{ki^*jyls} symbolizes the product of prices with quantities and is defined by the following constraints, which also have to be added to the problem:

$$0 \leq z_{ki^*jyls} \leq C^p b^p_{kyls}, \tag{27}$$

$$0 \leq q_{i^*jyls} - z_{ki^*jyls} \leq C^p(1 - b^p_{kyls}) \tag{28}$$

for C^p a suitably large constant.

IV. CASE STUDY

A. System Description

In this case study, we present a stylized electric power system consisting of three generation companies i_1, i_2, i_3, where i_1 will be the only firm deciding its investment in generation capacity while the capacity investments of the other two firms will be incorporated via three alternative scenarios s_1, s_2, s_3. It will be assumed that generation companies i_2 and i_3 are identical. The installed capacity of these two firms in the first year is given in Table I for each of the investment scenarios. The initial investments are assumed to increase 0.5% with every passing year.

Three different scenarios of the competitors' investments and corresponding conjectured price responses θ are incorporated: perfect competition (s_1), an intermediate case (s_2) which lies between perfect competition and the Cournot oligopoly, and finally the Cournot oligopoly (s_3). Table II provides the probability w_s as well as the corresponding conjectured price response θ of each scenario.

Company i_1 will have the choice between two different technologies, which correspond to Nuclear and CCGT, where Nuclear represents a base-load technology as it has high investment costs and low variable costs and CCGT represents a peak-load technology because it has lower investment but higher production costs. Investment costs and production costs of each technology are presented in Table III and were estimated based on data given in [24]. We assume these costs are the same for every company.

TABLE III
PRODUCTION AND INVESTMENT COSTS

Technology	Production Cost δ [€/MWh]	Investment Const β [€/MW/yr]
Nuclear	11.8	46,000
CCGT	22	15,000

TABLE IV
DEMAND INTERCEPT IN YEAR 1 [GW] AND ANNUAL LOAD LEVEL DURATIONS [H]

Demand Intercept		Load Level Durations	
peak	off-peak	peak	off-peak
55.213	36.036	3300	5460

TABLE V
TOTAL EXPECTED PROFITS OF INVESTING FIRM IN [M€]

	MPEC	MILP	MILP
		$\Delta_p = 0.15$ [€/MWh]	$\Delta_p = 0.04$ [€/MWh]
Profits	30921	30691	31877

Table IV provides the duration of each of the two load levels and the intercept of the demand curve in the first year. Each year, a sustained 3% increase of the demand is considered. The demand slope α was chosen to be 0.23 [GW/(€/MWh)] based on [25]. Moreover we consider a time scope of five years and a discount rate f of 2%.

B. Results

All models have been formulated in GAMS. The computational time to solve the stochastic MPEC of the case study—a problem of 643 variables—by initializing smartly and by using the solver CONOPT on an Intel(R) Core(TM) 2 Quad Processor with 3.21 GB of RAM was 0.8 s. Using CPLEX to solve the case study problem as a MILP with a step size $\Delta_{p_{yls}} = 0.15[€/\mathrm{MWh}]$ yields a model of about 1850 variables and took about 11.5 h. For a MILP with a finer step size of $\Delta_{p_{yls}} = 0.04[€/\mathrm{MWh}]$, the number of variables increases to about 2050 and the resolution time increases to about 20 h.

In Table V, we compare the expected profits of the investing firm obtained by solving the MPEC using the nonlinear solver CONOPT and by solving the MILP with two different step sizes using CPLEX. In this particular case, we observe that the MPEC yields expected profits that are higher than the profits obtained by the coarser MILP but lower than the profits of the finer MILP, implying that the choice of $\Delta_{p_{yls}}$ is critical.

In Fig. 1, we present the corresponding capacity investment decisions over the entire time horizon, as well as the starting point that was used to compute the MPEC, which is referred to as MCP and was obtained by solving a simultaneous optimization problem, as the MCP given in [11]. The upper plot depicts capacity investments in Nuclear and the lower plot depicts the capacity investments in CCGT. Comparing the investment decisions of the two MILPs, we observe that even though there only is a 4% difference in the objective function value, the investment decisions are very different, i.e., under the coarser MILP, there is more than double the investment in CCGT than under

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

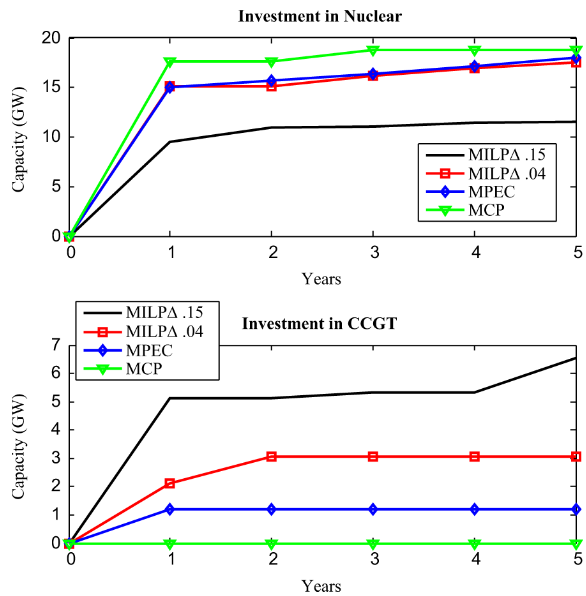

Fig. 1. Capacity investment results obtained by MILPs, MPEC, and MCP.

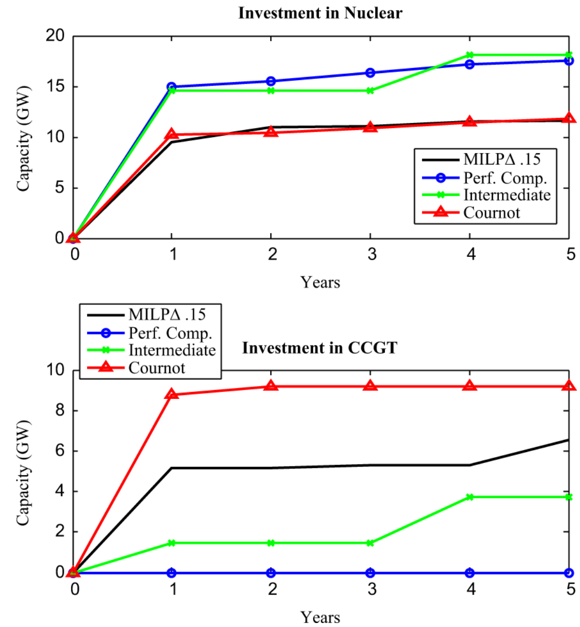

Fig. 2. Capacity investment results obtained by stochastic MILP and deterministic MILPs.

the finer MILP and there are 6 GW less of Nuclear investments. This shows that the solution of the MILP depends considerably on the choice of the step size and hence it has to be chosen carefully.

Moreover we observe that for this case study, the investment decisions of the MPEC are not too different from the decisions taken by the finer MILP, i.e., $\Delta_p = 0.04[€/\mathrm{MWh}]$, while only taking a small fraction of its computational time. In particular, when comparing the MPEC and the finer MILP, the investment in Nuclear is almost exactly the same—in the last year, there only is a difference of about 0.5 GW out of 18 GW installed. Under the finer MILP, the investment in CCGT is about 1.9 GW higher than under the MPEC. In terms of total capacity, the difference amounts to 1.4 GW, which yields a relative difference of only 7%.

Fig. 2 gives the investment results of the stochastic MILP with $\Delta_{p_{yls}} = 0.15[€/\mathrm{MWh}]$ and compares them to three deterministic MILPs, each representing one of the three scenarios individually. In particular, we solve three deterministic MILPs, each considering a fixed generation capacity of the competition, given by Table I, and a corresponding strategic behavior in the spot market (lower level), which is characterized by θ, given in Table II. As scenario s_1 assumes a perfectly competitive market, we will refer to its deterministic results as "Perf. Comp." The deterministic results of scenario s_2 will be referred to as "Intermediate" and the deterministic results of scenario s_3 shall be referred to as "Cournot" as the strategic behavior in the market is the Cournot oligopoly. Finally, note that each deterministic MILP was discretized using the same step size.

The first observation we make is that the investment in the stochastic case lies between the deterministic scenarios as expected. The stochastic investment in Nuclear almost exactly coincides with the Cournot solution, while the stochastic investment in CCGT lies about halfway between the Cournot and the Intermediate solution. We furthermore notice that when the strategic behavior in the spot market becomes more competitive, the investments in the peak load technology CCGT decrease while investments in the base load technology Nuclear seem to increase. This can be explained as follows. When the spot market is perfectly competitive, it is not lucrative for the investing agent to build peaking units because these units do not yield any profits in off-peak hours. In off-peak hours, capacity will not be binding in the market and hence the perfectly competitive solution in the market yields the market price as the marginal cost of the most expensive unit needed. Even if this unit is a CCGT peaking unit and is hence dispatched during off-peak hours, it would only recover its variable cost but not the investment cost. In peak hours, capacity will be binding and the market price will be higher than the marginal cost of a peaking unit as there is under-investment in capacity even in the perfectly competitive case. This is due to the fact that in a two-stage model the investing generation company is aware that investments influence market outcomes and that if there were over-investment there would be no profits at all. This is often observed for two-stage models. However, in this case study, the peak period seems to be too short for the peaking units to be profitable under perfect competition.

On the other hand, when the strategic behavior in the spot market is oligopolistic, then the market price will be above marginal costs, hence yielding profits for peaking units in both peak and off-peak periods. Therefore, investment in peaking units becomes more attractive under oligopolistic behavior than under perfect competition. Moreover, under oligopolistic behavior, the investing generation company can exert market power and hence raises prices by decreasing its amount of base load capacity. Both effects can be observed in Fig. 2.

V. Conclusions

We have introduced a stochastic bilevel model to assist a generation company to decide its investment schedule assuming different investment scenarios of the competition. In the upper level, the investing agent maximizes its expected net present value. The lower level corresponds to a conjectural variations market equilibrium, which allows us to represent strategic behavior from perfect competition, the Cournot oligopoly, and intermediate cases.

Two resolution methods are presented, a nonlinear method and a linearization method. The nonlinear method quickly resolves the exact problem but cannot guarantee global optimality. The linearization method guarantees a global optimum, but it takes much longer to solve and the choice of the step size is critical.

The proposed bilevel methodology has been applied to a case study, and results have been presented.

ACKNOWLEDGMENT

The authors would like to thank L. Barroso and Profs. R. Baldick and B. F. Hobbs for their comments and support.

REFERENCES

[1] H. M. Merrill and F. C. Schweppe, "Strategic planning for electric utilities: Problems and analytic methods," *Interfaces*, vol. 14, no. 1, pp. 72–83, Jan. 1984.

[2] T. H. Lee, B. C. Ball, and R. D. Tabors, "Energy aftermath (books and reports)," *IEEE Power Eng. Rev.*, vol. 11, no. 6, p. 45, Jun. 1991.

[3] J. J. Śanchez, "Strategic analysis of the long-term planning of electric generation capacity in liberalised electricity markets," Ph.D. dissertation, Univ. Pontificia Comillas de Madrid, Madrid, Spain, 2009.

[4] F. Facchinei and J.-S. Pang, *Finite Dimensional Variational Inequalities and Complementarity Problems*. New York: Springer-Verlag, 2003, vol. 1.

[5] R. Cottle, J.-S. Pang, and R. Stone, *The Linear Complementarity Problem*. San Diego, CA: Academic, 1992.

[6] B. F. Hobbs, C. B. Metzler, and J.-S. Pang, "Calculating equilibria in imperfectly competitive power markets: An MPEC approach," *IEEE Trans. Power Syst.*, vol. 15, no. 2, pp. 638–645, May 2000.

[7] A. Ramos, M. Ventosa, and M. Rivier, "Modeling competition in electric energy markets by equilibrium constraints," *Util. Pol.*, vol. 7, pp. 233–242, 1998.

[8] C.-L. Su, "Equilibrium problems with equilibrium constraints: Stationarities, algorithms and applications," Ph.D. dissertation, Stanford Univ., Stanford, CA, 2005.

[9] D. Ralph and Y. Smeers, "EPECs as models for electricity markets," in *Proc. IEEE PES Power Systems Conf. Expo. (PSCE '06)*, 2006, pp. 74–80.

[10] F. H. Murphy and Y. Smeers, "Generation capacity expansion in imperfectly competitive restructured electricity markets," *Oper. Res.*, vol. 53, no. 4, pp. 646–661, Jul. 2005.

[11] M. Ventosa, R. Denis, and C. Redondo, "Expansion planning in electricity markets. Two different approaches," in *Proc. 14th Power Systems Computation Conf.*, Jul. 1, 2003, pp. 1–8.

[12] E. Centeno, J. Reneses, R. Garcia, and J. J. Sanchez, "Long-term market equilibrium modeling for generation expansion planning," in *Proc. IEEE Bologna Power Tech Conf.*, 2003, vol. 1.

[13] A. Haurie, R. Loulou, and G. Savard, "A two-level systems analysis model of power cogeneration under asymmetric pricing," in *Proc. Amer. Control Conf.*, 1990, pp. 2095–2099.

[14] R. Garcia-Bertrand, D. Kirschen, and A. J. Conejo, "Optimal investments in generation capacity under uncertainty," in *Proc. 16th Power Systems Computation Conf.*, Jul. 14–18, 2008, pp. 1–7. [Online]. Available: http://www.pscc2008.com.

[15] S. J. Kazempour, A. J. Conejo, and C. Ruiz, "Strategic generation investment using a complementarity approach," *IEEE Trans. Power Syst.*, vol. 26, no. 2, pp. 940–948, May 2011.

[16] E. Centeno, J. Reneses, and J. Barquin, "Strategic analysis of electricity markets under uncertainty: A conjectured-price-response approach," *IEEE Trans. Power Syst.*, vol. 22, no. 1, pp. 423–432, Feb. 2007.

[17] J. Barquin, E. Centeno, and J. Reneses, "Medium-term generation programming in competitive environments: A new optimisation approach for market equilibrium computing," *Proc. Inst. Elect. Eng., Gen., Transm., Distrib.*, vol. 151, no. 1, pp. 119–126, Jan. 2004.

[18] J. Reneses, E. Centeno, and J. Barquin, "Medium-term marginal costs in competitive generation power markets," *Proc. Inst. Elect. Eng., Gen., Transm., Distrib.*, vol. 151, no. 5, pp. 604–610, Sep. 2004.

[19] S. L. de Haro, P. Sánchez, J. de la Hoz Ardiz, and J. Fernández, "Estimating conjectural variations for electricity market models," *Eur. J. Oper. Res.*, vol. 181, no. 3, pp. 1322–1338, 1983.

[20] D. W. Bunn, *Modelling Prices in Competitive Electricity Markets*. New York: Wiley, 2004.

[21] C. A. Diaz, J. Villar, F. A. Campos, and J. Reneses, "Electricity market equilibrium based on conjectural variations," *Elect. Power Syst. Res.*, vol. 80, no. 12, pp. 1572–1579, Dec. 2010.

[22] J. Fortuny-Amat and B. McCarl, "A representation and economic interpretation of a two-level programming problem," *J. Oper. Res. Soc*, vol. 32, no. 9, pp. 783–792, 1981.

[23] M. V. Pereira, S. Granville, M. H. C. Fampa, R. Dix, and L. A. Barroso, "Strategic bidding under uncertainty: A binary expansion approach," *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 180–188, Feb. 2005.

[24] I. E. A Neclear Energy Agency, *Projected Costs of Generating Electricity. 2005 Update,* OECD Publications, 2005.

[25] A. Garcia-Alcalde, M. Ventosa, M. Rivier, A. Ramos, and G. Relao, "Fitting electricity market models. A conjectural variations approach," in *Proc. 14th Power Systems Computation Conf.*, Jun. 2002, p. 6, Session 12, Paper 3.

Sonja Wogrin received the technical mathematics degree from the Graz University of Technology, Graz, Austria, in 2008 and the M.S. degree in computation for design and optimization from the Massachusetts Institute of Technology, Cambridge, MA, in 2008. She is currently pursuing the Ph.D. degree at the Instituto de Investigación Tecnológica, Comillas Pontifical University, Madrid, Spain.

Her research interests lie within the area of decision support systems in the energy sector and optimization.

Efraim Centeno received the industrial engineering degree and the Ph.D. degree in industrial engineering from the Comillas Pontifical University, Madrid, Spain, in 1991 and 1998, respectively.

He belongs to the research staff at the Instituto de Investigación Tecnológica, Comillas Pontifical University. His areas of interest include the planning and development of electric energy systems.

Julián Barquín (M'09) received the electrical engineering degree from the Comillas Pontifical University, Madrid, Spain, in 1988, the physics degree at UNED University in 1994, and the Ph.D. degree at the Comillas Pontifical University in 1993.

Since then, he has worked at the Comillas Pontifical University, where he is presently a Professor. His research activities have revolved around technical and economic analysis of energy systems.